8th Grade Texas Mathematics: Unpacked Content

What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?

Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

At A Glance:

New to Grade:

- (8.2A) Extending previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers.
- (8.2B) All square roots less than 225. Approximate and locate irrational numbers on a number line.
- (8.2C) Use of negative exponents in scientific notation.
- (8.2D) Order real numbers (changed from ordering rational numbers)
- (8.3A) Generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation. (new 7th grade)
- (8.3C) Use of algebraic representation to show scale factor (more formal language)
- (8.4A) Use of similar right triangles in exploring slope (Algebra I)
- (8.4B) Graph proportional relationships interpreting the unit rate with slope (Algebra I)
- (8.4C) Use of tables or graphs to determine rate of change and y intercept (Algebra I)
- (8.5A,B) Represent linear proportional and non-proportional situations with tables graphs and equations in the forms of $y=kx$ and $y=mx+b$ (Algebra I)
- (8.5C) Use of the term bivariate sets of data as opposed to scatterplots (Algebra I)
- (8.5D) Use a trend line to approximate the linear relationship between bivariate sets of data (aka line of best fit) (current TEKS and Algebra)
- (8.5E) Solve problems using direct variation (current TEKS, but with Algebra I terminology)
- (8.5F) Distinguish between proportional and non-proportional relations using $y=kx$ or $y=mx+b$ (current TEKS, more specific with equations)
- (8.5G) Identify functions using ordered pairs, tables, mappings, and graphs (Algebra I)
- (8.5I) Write an equation in the form of $y=mx+b$ (Algebra I)
- (8.7D) Determine the distance between two points on a coordinate plane using the Pythagorean Theorem. (Geometry)
- (8.8A,B,C) Write, model, and solve one variable equations or inequalities (from real world situations) with variables on both sides using rational number coefficients and constants (Algebra I)

- (8.8D) Use equations to establish facts about angle sum and exterior angles of triangles as well as angles created when parallel lines are cut by a transversal. (Geometry)
- (8.9) Identify solution (x,y) to two linear equations (system of equations) from the intersection of graphed equations. (Algebra I)
- (8.10A) Rotations (limited to multiples of 90 degrees through 360) on a coordinate plane (Geometry)
- (8.10C) Use an algebraic representation to explain the effects of translations, rotations, and reflections of two dimensional shapes on a coordinate plane. (Geometry)
- (8.11A) Construct a scatterplot and describe the data to address questions of association such as linear, nonlinear, and no association between bivariate data. (Algebra I)
- (8.11B) Determine the mean absolute deviation and the quantity as a measure of the average distance data are from the mean using a data set no more than 10 data points. (Statistics)
- (8.12) Personal Financial Literacy

Moved from Grade:

- The effect of scale factor on surface area and volume. (Geometry)
- Probability (7th grade)
- Compare rationals (6th grade)
- Order rationals is now order real numbers
- Solve problems with rational numbers in a variety of forms (deleted TEK, but embedded in Process Standards)
- Box and whisker plots (7th grade)
- Find and evaluate an algebraic expression to determine any term in an arithmetic sequence (embedded into 6th grade), but skills addressed in 8.4C.
- Draw 3-D figures from different perspectives
- Volume of prisms (other than cylinders) and pyramids. (7th grade)
- Surface area of pyramids (7th grade)
- Locate and name points on a coordinate plane using ordered pairs of rational numbers (Grade 6)
- Use of variability and measures of central tendency (6th grade)

- Select and use appropriate representation for displaying relationships among collected data (ie line plot, circle graph, bar graphs, box and whisker, etc) (6th grade and 7th grade)
- Recognize misuses of graphical representation and evaluate predictions (deleted, but skills are in the Process Standards)

Instructional Implications for 2013-14:

Due to the amount of material being moved from Algebra 1 down to both 7th and 8th grade, it will be important for middle school and high school teachers to collaborate on vertical alignment and the sharing of resources.

It would be an easier transition for teachers and students if teachers would begin implementation of some of the new TEKS in the 2013-14 school year.

Professional Learning Implications for 2013-14:

- PD and resources regarding Personal Financial Literacy
- PD related to the Linear Equations – High School teachers could mentor
- Teachers will need to identify the gaps that will need to be addressed in the 2013-14 school year.
- Embed the process standards into instruction and application
- Identify academic vocabulary
- Initial learning of the teachers' grade level TEKS (teachers unpacking the TEKS at their grade level)
- Vertical study of the strands to know how the TEKS align and progress from 7th through Algebra I

Grade 8 Primary Focal Areas:

The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency,and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) The primary focal areas in Grade 8 are proportionality; expressions, equations, relationships, and foundations of functions; and measurement and data. Students use concepts, algorithms, and properties of real numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students begin to develop an understanding of functional relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra

readiness skills necessitates the implementation of graphing technology.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

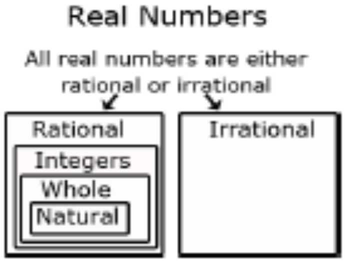
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

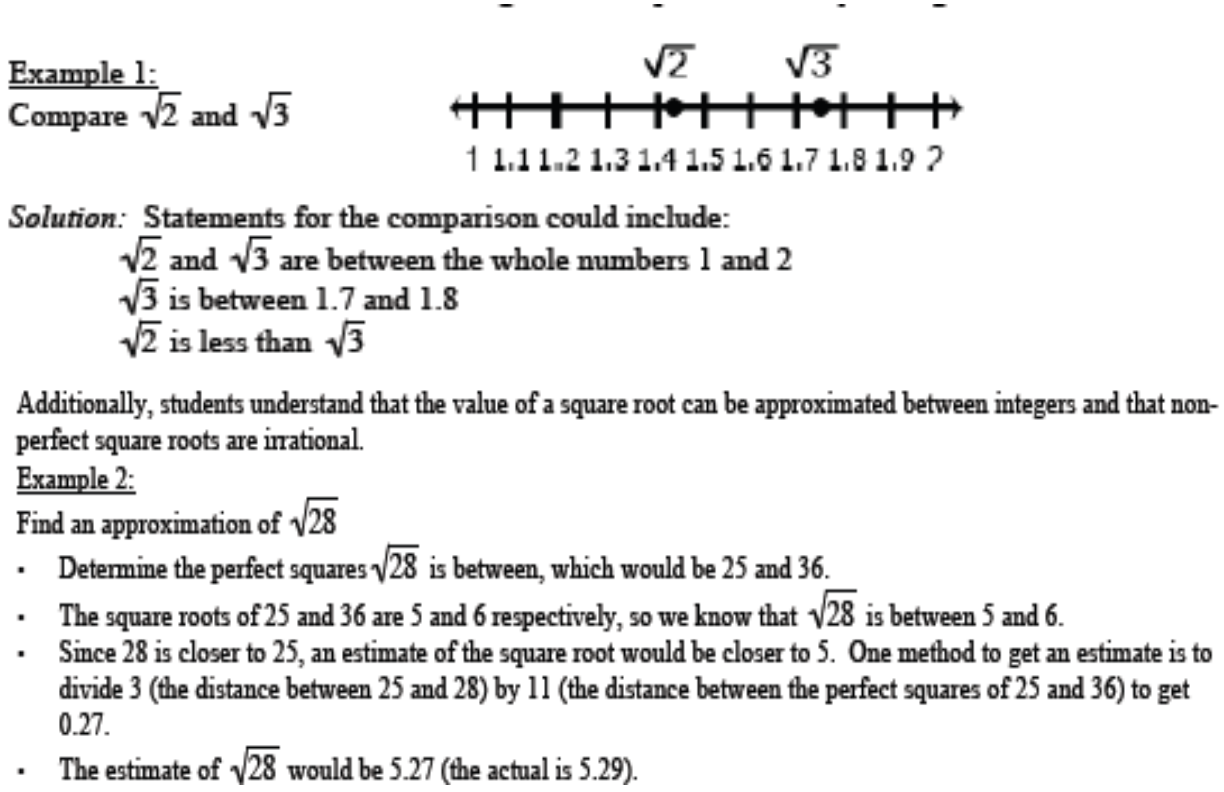
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Number and Operations: TEK 8.2	**The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:**
8.2(A) Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers	Students understand the relationship between the set of real numbers and the subsets that exist within the set. Students understand organization of the subsets: ie. natural numbers are a subset of whole numbers which is a subset of integers, which is a subset of rationals, which is a subset of the real numbers. Include the terminology: sometimes, always, never, and, and or when classifying numbers. Example: ½ is never an integer.. **Common errors:** 1. Students think that a number can only belong to one set such as integer. **Examples:**  2. Name all the sets the following numbers belong to: 3, 5.7, -8, $\sqrt{5}$, 4.23235......, π, $\sqrt{9}$, $\frac{4}{2}$
8.2(B) Approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line;	Students understand that an irrational number cannot be written in the form a/b. It is a non-terminating, non-repeating decimal. Students should know the perfect squares (1 to 15) in order to approximate the value of irrational numbers. **Common Misconception.** 1. Students think square root is dividing by 2.

	Examples:  Example 1: Compare $\sqrt{2}$ and $\sqrt{3}$ *Solution:* Statements for the comparison could include: $\sqrt{2}$ and $\sqrt{3}$ are between the whole numbers 1 and 2 $\sqrt{3}$ is between 1.7 and 1.8 $\sqrt{2}$ is less than $\sqrt{3}$ Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational. Example 2: Find an approximation of $\sqrt{28}$ - Determine the perfect squares $\sqrt{28}$ is between, which would be 25 and 36. - The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6. - Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27. - The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).
8.2(C) Convert between standard decimal notation and scientific notation; and	Students should understand the relationship between place value system and the powers of 10 used in scientific notation. ie: hundreds place is also 10^2. Students should understand that for a number to be in proper scientific notation it must be a number between one and ten multiplied by a power of 10. Students should understand that a negative power of 10 does not imply a negative number but a number less than one, since a negative exponent means division by a power of 10.

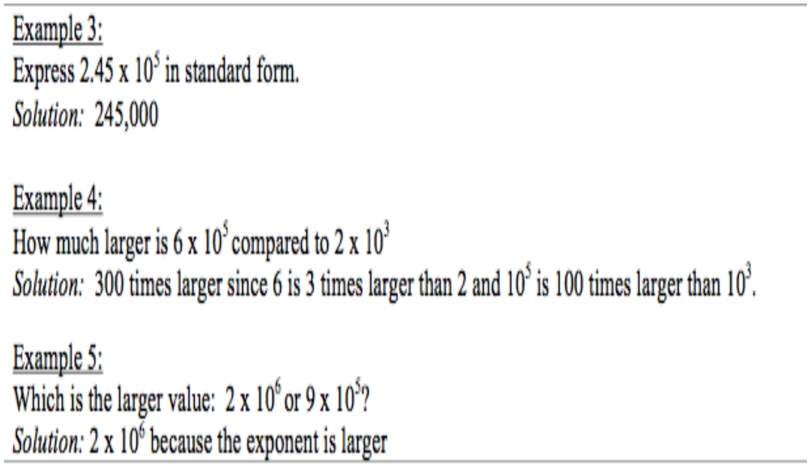
Examples:

Express 0.00045 in scientific notation:

$4.5x10^{-4}$

Express 32,000,000 in scientific notation:

$3.2x10^{6}$



Example 3:

Express 2.45 x 10^{5} in standard form.

Solution: 245,000

Example 4:

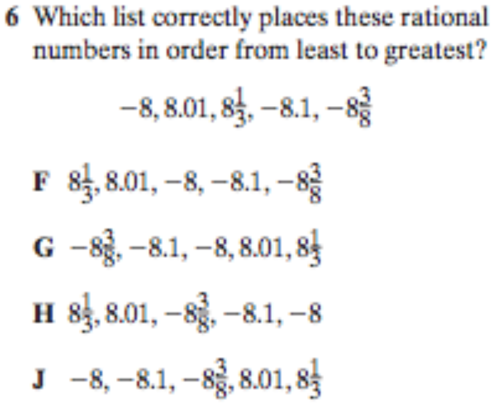
How much larger is 6 x 10^{5} compared to 2 x 10^{3}

Solution: 300 times larger since 6 is 3 times larger than 2 and 10^{5} is 100 times larger than 10^{3}.

Example 5:

Which is the larger value: 2 x 10^{6} or 9 x 10^{5}?

Solution: 2 x 10^{6} because the exponent is larger

8.2(D) Order a set of real numbers arising from mathematical and real-world contexts.	Students understand the value of numbers in various forms (decimals, fractions, percents), how to standardize them into one format (such as converting all to decimals) and how to use their values to place them in correct order (both ascending and descending). Students understand a number line and the ordering of both negative and positive numbers. Students understand that irrational numbers are a part of the real number system. **Common Misconceptions** 1. Students need other vocabulary besides least to greatest such as fastest to slowest or thickest to thinnest. Students automatically look for the smallest number to be the smallest. **Examples:**  **6** Which list correctly places these rational numbers in order from least to greatest? $-8, 8.01, 8\frac{1}{3}, -8.1, -8\frac{3}{8}$ **F** $8\frac{1}{3}, 8.01, -8, -8.1, -8\frac{3}{8}$ **G** $-8\frac{3}{8}, -8.1, -8, 8.01, 8\frac{1}{3}$ **H** $8\frac{1}{3}, 8.01, -8\frac{3}{8}, -8.1, -8$ **J** $-8, -8.1, -8\frac{3}{8}, 8.01, 8\frac{1}{3}$ The following represent finish times in a race. Order the times from fastest to slowest. 43.5 sec., 41.6 sec., 42.1 sec., 42.8 sec.

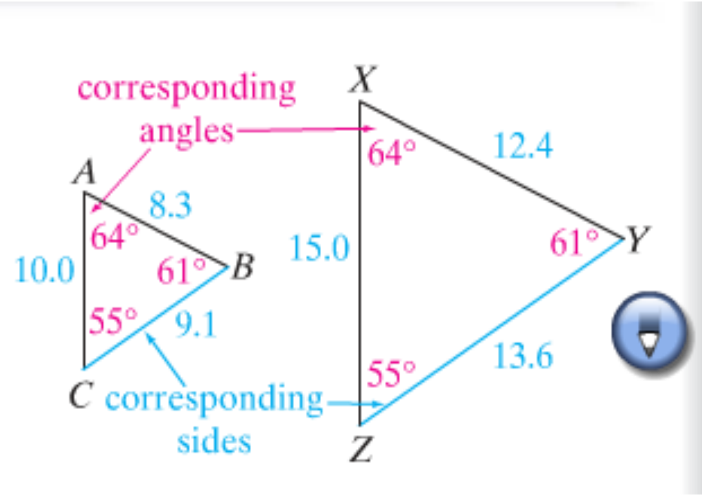
Several stores are having sales. The prices are reduced by 62.5%, $\frac{2}{3}$, 75%, $\frac{1}{2}$, and $\frac{7}{10}$. Which list shows the price reductions from greatest to least?

A 75%, 62.5%, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{10}$

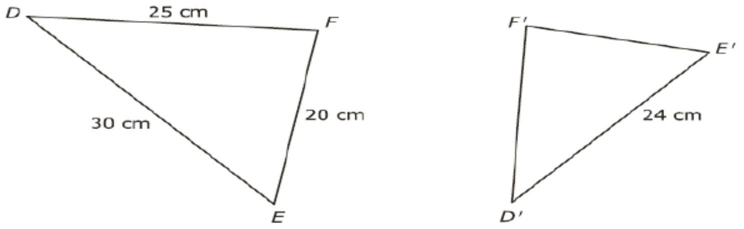
B 75%, $\frac{7}{10}$, $\frac{2}{3}$, 62.5%, $\frac{1}{2}$

C 75%, $\frac{7}{10}$, 62.5%, $\frac{1}{2}$, $\frac{2}{3}$

D 75%, $\frac{7}{10}$, 62.5%, $\frac{2}{3}$, $\frac{1}{2}$

Proportionality: **TEK 8.3**	**The student applies mathematical process standards to use proportional relationships to describe dilations.**
8.3(A) Generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation;	The student understands that the lengths of corresponding sides are in proportion. Students understand that similar figures are produced from dilations. Students need to know prime notation to indicate the dilated figure from the original. **Common Misconceptions:** 1. Not matching corresponding parts. Need to use the name and not the figure. 2. Not all images are drawn to scale. **Examples:** $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$ The symbol ~ means *is similar to.* At the right, $\triangle ABC \sim \triangle XYZ$. 

Triangle *DEF* was dilated to create triangle *D′E′F′*.



What is the length of segment *E′F′* ?

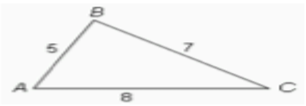
A 14 cm

B 20 cm

[✓ C] 16 cm

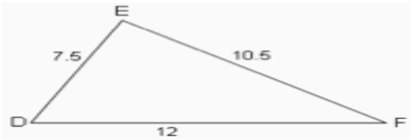
D 18 cm

Triangle *ABC* is shown below.

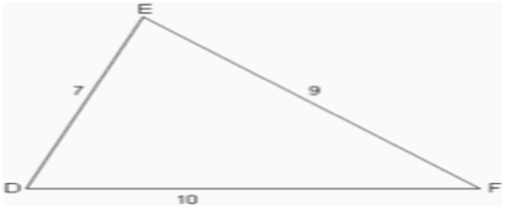


Which of the triangles below is NOT similar to ΔABC ?

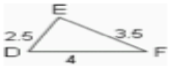
A



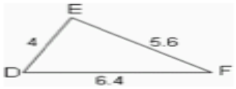
B

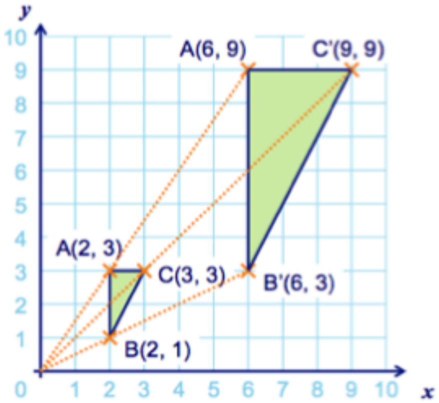


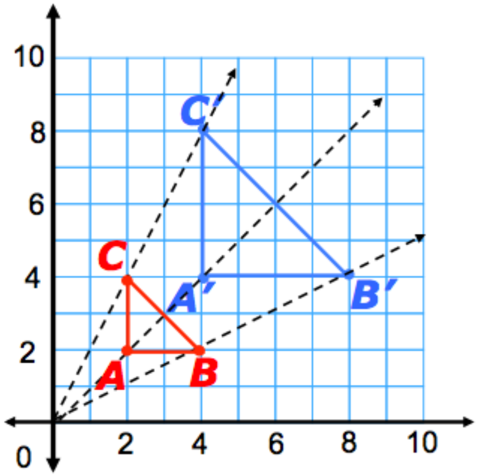
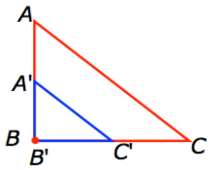
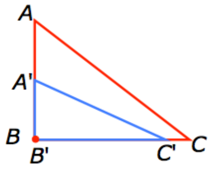
C



D

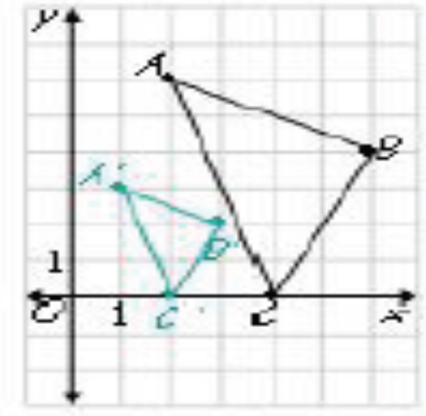


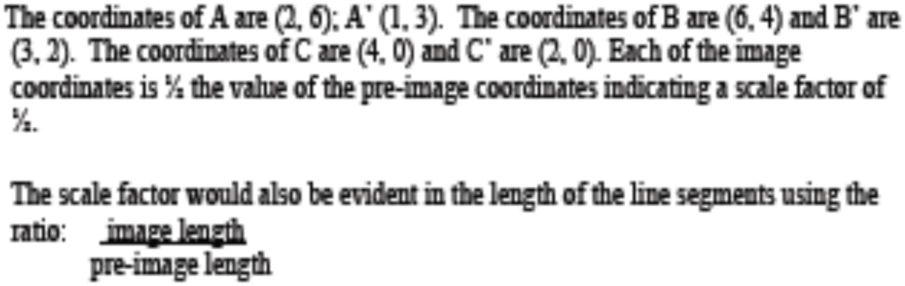
8.3(B) Compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and	Students understand that similar figures have congruent angles and sides that are proportional. Students understand that similar figures are produced from dilations. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size. **Common Misconceptions.** 1. Students think that the scale factor also applies to the angle measure. 2. Students do not match corresponding sides to show sides are proportional. **Examples:** 

	 **Tell whether each transformation is a dilation.** **C.**  The transformation is a dilation. **D.**  The transformation is not a dilation. The figure is distorted.
8.3(C) Use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation	Students understand that scale factor shows the multiplicative relationship between the dimensions of two figures. Students understand that the scale factor can also show the multiplicative relationship between coordinates of corresponding points on a figure if the figures have the origin as their center of dilation. Students understand that the algebraic form: (x,y)→(ax,ay) will provide the new coordinates of the dilation where ‘a’ is the scale factor when the image is dilated about the origin. Students must have knowledge of correct vocabulary. For example, no longer using enlargement/reduction, now using “dilation with a positive rational scale factor.”

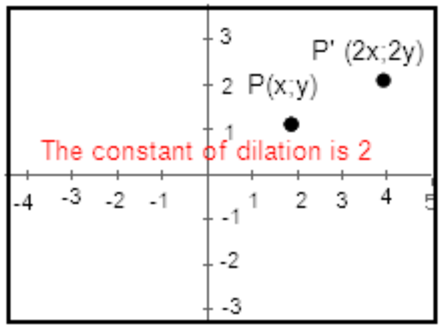
Examples:

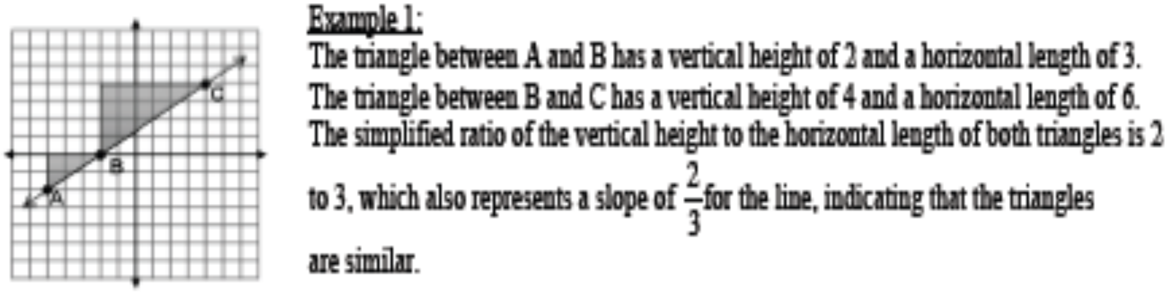
Example 1: In the below image the algebraic representation would be (x,y) → (1/2x, 1/2y)




The coordinates of A are (2, 6); A' (1, 3). The coordinates of B are (6, 4) and B' are (3, 2). The coordinates of C are (4, 0) and C' are (2, 0). Each of the image coordinates is ½ the value of the pre-image coordinates indicating a scale factor of ½.

The scale factor would also be evident in the length of the line segments using the ratio: $\frac{\text{image length}}{\text{pre-image length}}$

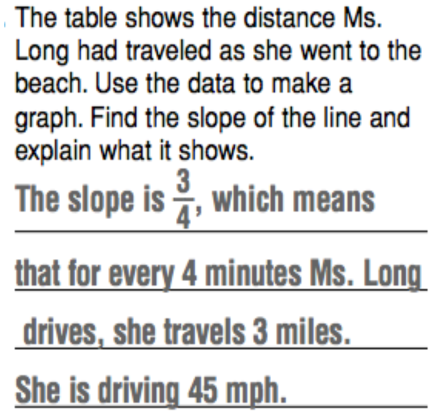
	The image of P(x,y) under a dilation from the origin is P'(cx,cy), where c is the constant of dilation, so c does not equal 0. 

Proportionality: **TEK 8.4**	**The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope.**
8.4(A) Use similar right triangles to develop an understanding that slope, *m*, given as the rate comparing the change in *y*-values to the change in *x*-values, (*y*2 - *y*1) / (*x*2 - *x*1), is the same for any two points (*x*1, *y*1) and (*x*2, *y*2) on the same line	Students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line. Students understand that slope, *m*, compares the change in any y-values to the change in x-values. $(y_2 - y_1) / (x_2 - x_1)$ when (x_1, y_1) and (x_2, y_2) are points on the same line. **Common Errors:** 1. Students will put change in x-values to the change in y-values. 2. Students may use $(y_1 - y_2) / (x_2 - x_1)$ where the reverse the order of the points. **Examples:**  Example 1: The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $\frac{2}{3}$ for the line, indicating that the triangles are similar.
8.4(B) Graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship	Student understands that the graph of a proportional relationship will always have a y-intercept (intersect with) the origin (0, 0). Student understands that the unit rate in a given situation will become the slope of their graph; student will graph one point at (0, 0) and another at (1, y), where y is their unit rate and extend the line through future points (2, 2y), (3, 3y).

Common Misconceptions

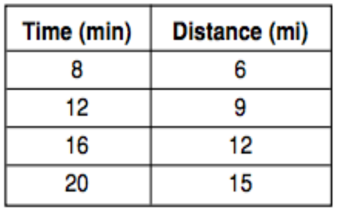
1. Unit rate is amount per one unit, Slope isn't always a whole number.

Examples:

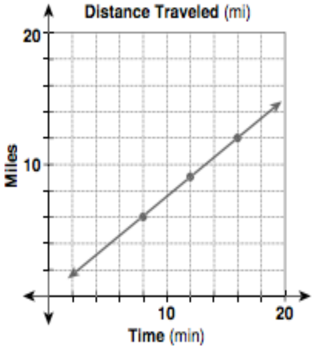


The table shows the distance Ms. Long had traveled as she went to the beach. Use the data to make a graph. Find the slope of the line and explain what it shows.

The slope is $\frac{3}{4}$, which means that for every 4 minutes Ms. Long drives, she travels 3 miles. She is driving 45 mph.



Time (min)	Distance (mi)
8	6
12	9
16	12
20	15

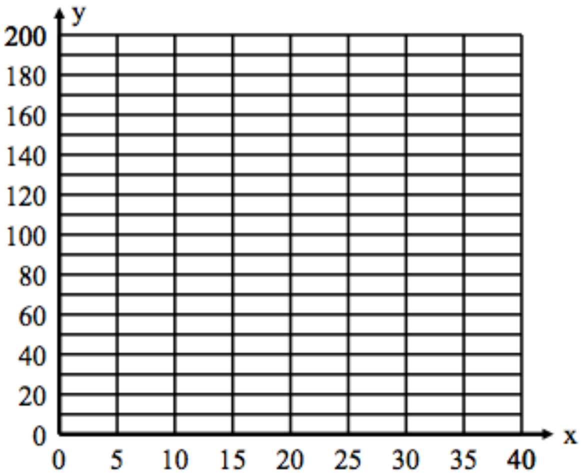


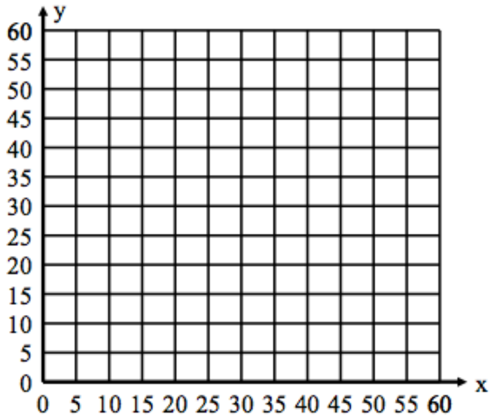
4 . Rosa earns $5 per hour for babysitting.

Hours # (x)	Earnings(y)

Rate of change ___

Slope of the line ___





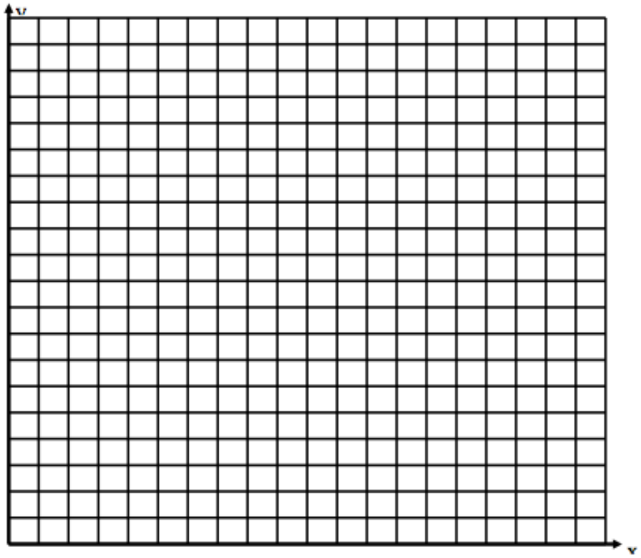
A phone card rate is .05/min.

Run (x)	Rise (y)

Rate of change ___________

Slope of the line ___________

5. Think of a relational situation in your life that changes at a steady rate. Create a table showing five ordered pairs. Write the rate as the slope of the line, and graph the linear relation.



6. Explain how the slope of the line compares to the rate of change for the data.

Extra example problems and answers

Problems

For each of the following problems, draw the graph of the proportional relationship between the two quantities and describe how the unit rate is represented on the graph.

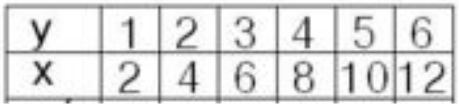
1. An Elm tree grows 8 inches each year.
2. Davis adds $3.00 to his savings account each week.
3. Bananas are $2.40 per pound.
4. Lunches in the cafeteria are $2.25 each.

	Answers 1. The graph of y = 8x , which is a line passing through (0, 0) with a slope of 8; the slope 8 is the rate of change of the tree each year. 2. The graph of y = 3x , which is a line passing through (0, 0) with a slope of 3; the slope 3 is the rate of change of Davis' account each week. 3. The graph of y = 2.4x , which is a line passing through (0, 0) with a slope of 2.4; the slope 2.4 is the unit rate of each pound of bananas. 4. The graph of y = 2.25x , which is a line passing through (0, 0) with a slope of 2.25; the slope 2.25 is the unit rate of change of each lunch.
8.4(C) Use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems.	Student understands that the graph of a proportional relationship will always have a y-intercept (intersect with) the origin (0, 0). Student determines the rate of change (slope) from a proportional graph by finding the value of the y-coordinate when the x-coordinate is one. Student determines the rate of change (slope) from a proportional table by finding the value of the output when the input is Student understands that slope, rate of change, and unit rate mean the same thing. Student understands that in a non-proportional linear relationship, the y-intercept will not be at (0,0). **Common Misconceptions.** 1. Students think that constant rate of change automatically means it is proportional.

Examples:

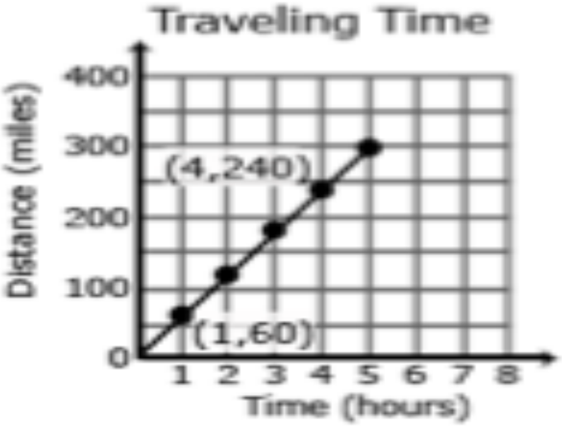
In this table, the rate of change is 2 since 8/4, 6/3 etc.

:



y	1	2	3	4	5	6
x	2	4	6	8	10	12

All have the same value in this graph, the rate of change would be 60 since 240/4 and 60/1 have the same value.



Word Problem:

Reagan has $39 in her savings. She adds $32 to it each month. Write an equation to show how much she has each month as a function of the deposits.

Equation:

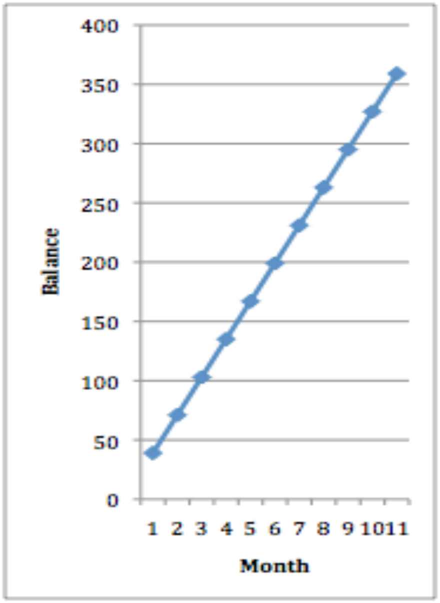
M= number of months
B= balance

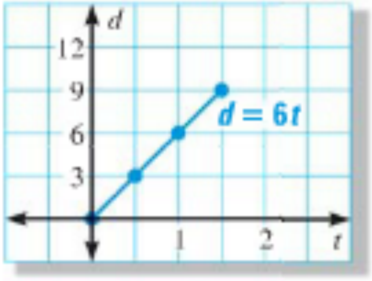
B= 39 + 32m

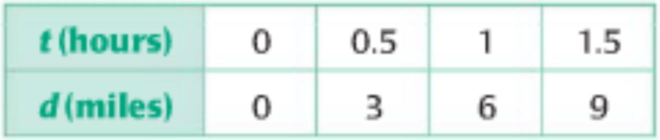
Table:

m	B= 39 + 32m	b
0	39 + 32(0)	39
1	39 + 32(1)	71
2	39 + 32(2)	103
3	39 + 32(3)	135
4	39 + 32(4)	167
5	39 + 32(5)	199
6	39 + 32(6)	231
7	39 + 32(7)	263
8	39 + 32(8)	295
9	39 + 32(9)	327
10	39 + 32(10)	359

Graph:



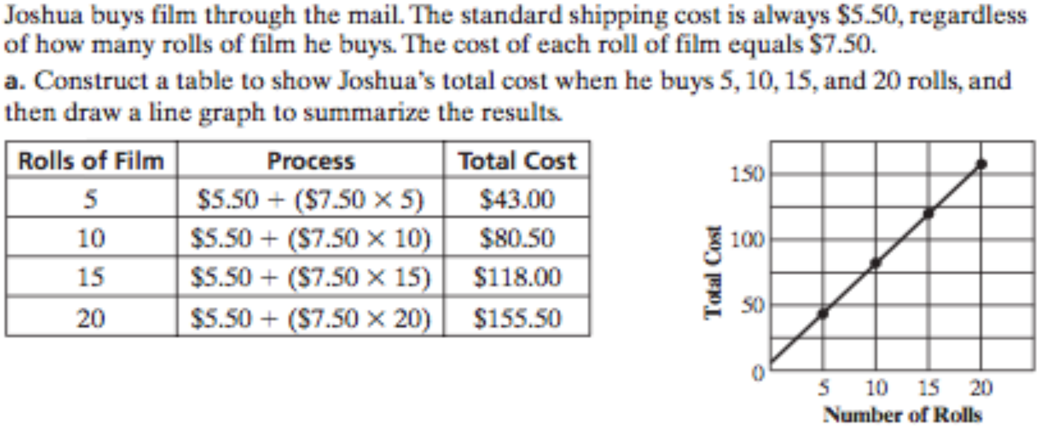
<table>
<tr><th>Proportionality:
TEK 8.5</th><th>The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.</th></tr>
<tr><td>8.5(A) Represent linear proportional situations with tables, graphs, and equations in the form of <i>y</i> = <i>kx</i>;</td><td>
<p>Students understand from 7th grade that k=y/x is the constant of proportionality. We are now also referring to k as slope, rate of change, and unit rate. Student understands that the graph must go through (0,0).</p>
<p>Common misconceptions.</p>
<p>Students may calculate slope (k) as x/y instead of y/x.</p>
<p>Examples:</p>
<p>Example 1: Susan is able to bike 18 miles in 3 hours. Create a table, graph, and equation to represent any distance if her rate stays constant.</p>



<table>
<tr><td>t (hours)</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td></tr>
<tr><td>d (miles)</td><td>0</td><td>3</td><td>6</td><td>9</td></tr>
</table>
</td></tr>
<tr><td>8.5(B) Represent linear non-proportional situations with tables, graphs, and equations in the form of <i>y</i> = <i>mx</i> + <i>b</i>, where $b \neq 0$;</td><td>Student understands that linear situations can be represented four ways: in words, tables, graphs and as an equation. Student understands that non-proportional situations will affect the graph such that the y-intercept will no longer be zero; it will now be the number added or subtracted (b). Student understands that non-proportional situations affect the equation such that there will be a number added or subtracted (b). Student understands that the ratios represented in the table will no longer be equivalent (not proportional).</td></tr>
</table>

Common Misconceptions.

If graphing using slope intercept form, students may use m as the intercept and b as the slope.

Examples:



Joshua buys film through the mail. The standard shipping cost is always $5.50, regardless of how many rolls of film he buys. The cost of each roll of film equals $7.50.

a. Construct a table to show Joshua's total cost when he buys 5, 10, 15, and 20 rolls, and then draw a line graph to summarize the results.

Rolls of Film	Process	Total Cost
5	$5.50 + ($7.50 × 5)	$43.00
10	$5.50 + ($7.50 × 10)	$80.50
15	$5.50 + ($7.50 × 15)	$118.00
20	$5.50 + ($7.50 × 20)	$155.50

8.5(C) Contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation;

Students will be able to look at graphs and determine, based on the y-intercept (b), whether it shows proportionality (b = 0) or not ($b \neq 0$). Student understands that bivariate means “two variables”, in other words there are two types of data.

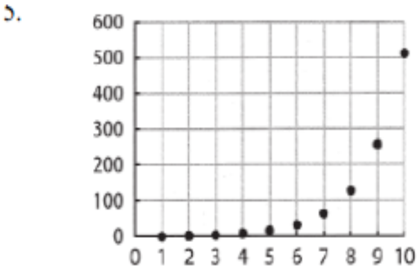
Common Misconceptions.

Students may think that if the graph is nonlinear, then there is no relationship. (ie. exponential functions)

Examples:

Examples of non linear relationships

X	Y
1	12
2	7
3	4
4	3
5	4
6	7



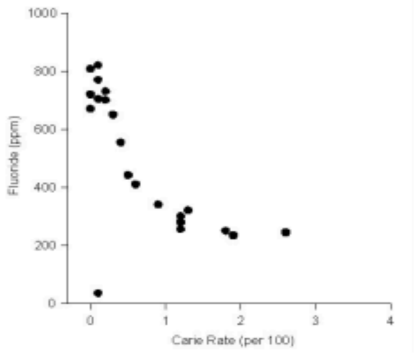
Example 4:
The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

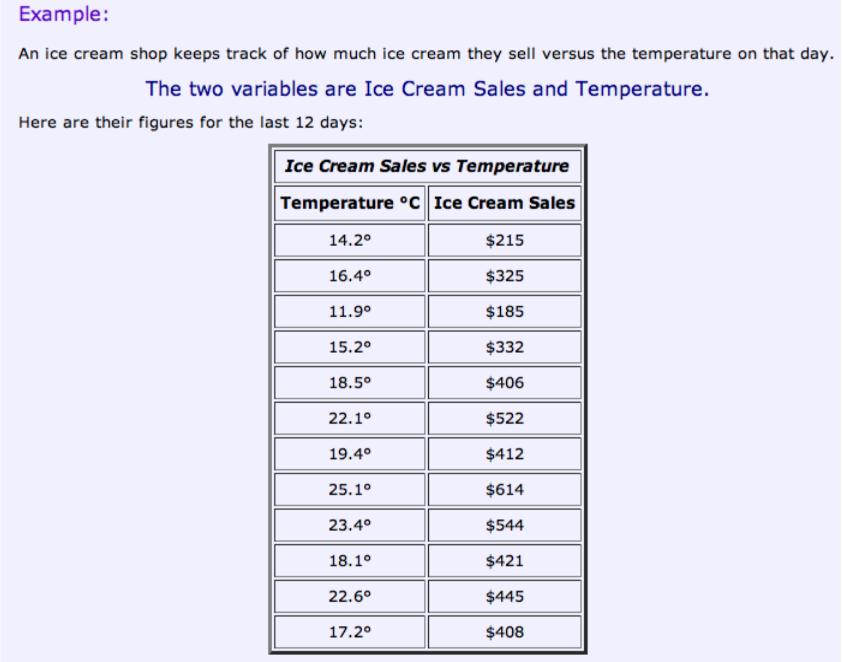
Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Solution: There is a positive association.

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model.
NOTE: Use of the formula to identify outliers is **not** expected at this level.

Students recognize that not all data will have a linear association. Some associations will be non-linear as in the example below:





Example:

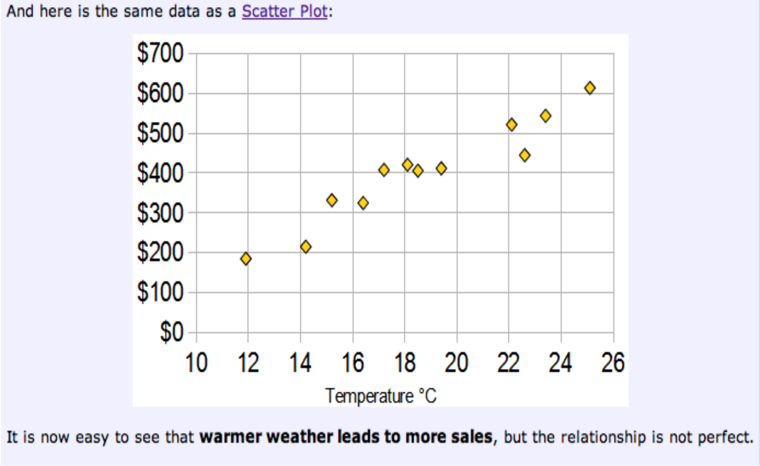
An ice cream shop keeps track of how much ice cream they sell versus the temperature on that day.

The two variables are Ice Cream Sales and Temperature.

Here are their figures for the last 12 days:

Ice Cream Sales vs Temperature

Temperature °C	**Ice Cream Sales**
14.2°	$215
16.4°	$325
11.9°	$185
15.2°	$332
18.5°	$406
22.1°	$522
19.4°	$412
25.1°	$614
23.4°	$544
18.1°	$421
22.6°	$445
17.2°	$408



And here is the same data as a Scatter Plot:

It is now easy to see that **warmer weather leads to more sales**, but the relationship is not perfect.

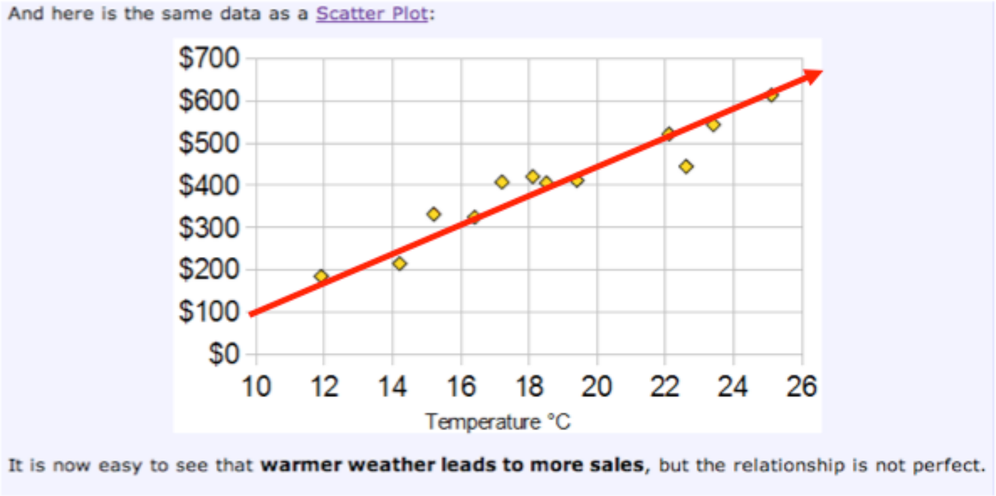
8.5(D) Use a trend line that approximates the linear relationship between bivariate sets of data to make predictions;

Student understands that trend line and line of best fit mean the same thing. Student understands that the trend line closely follows the path of points passing through as many as possible with about half the remaining points both above and below the line.

Common Misconceptions:

1. Students try to hit every point rather than through the middle.
2. Trend line does not have to start at the origin.

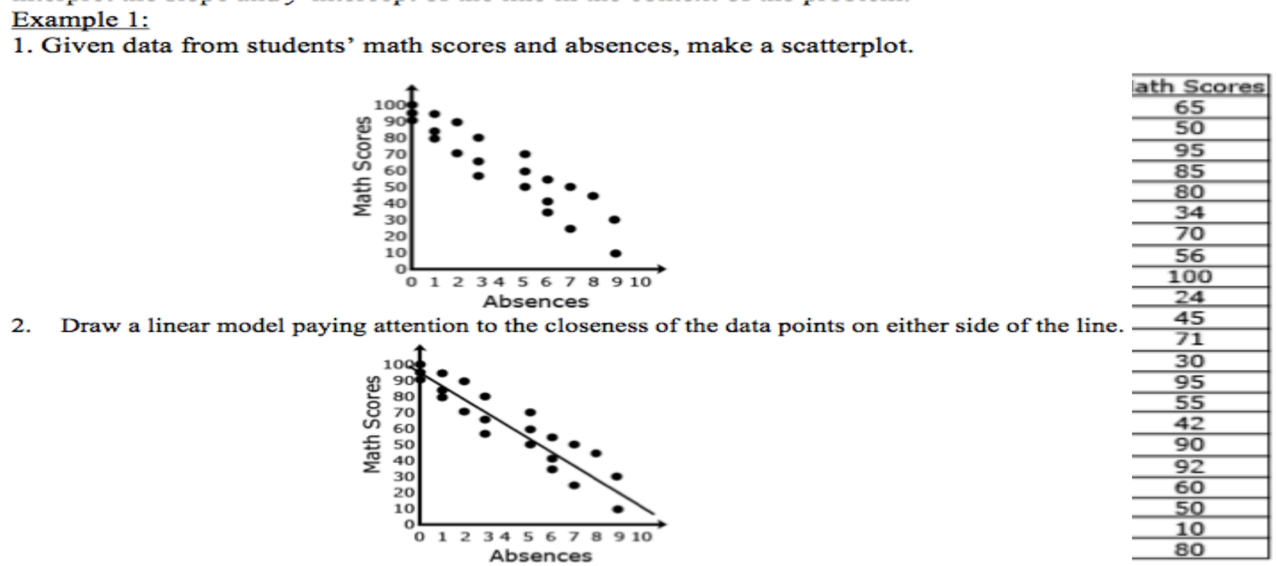
Examples:



And here is the same data as a Scatter Plot:

It is now easy to see that **warmer weather leads to more sales**, but the relationship is not perfect.

<table>
<tr>
<td></td>
<td>
Linear models can be represented with a linear equation. Students interpret the slop and y-intercept of the line in the context of the problem.



<u>Example 1:</u>

1. Given data from students’ math scores and absences, make a scatterplot.

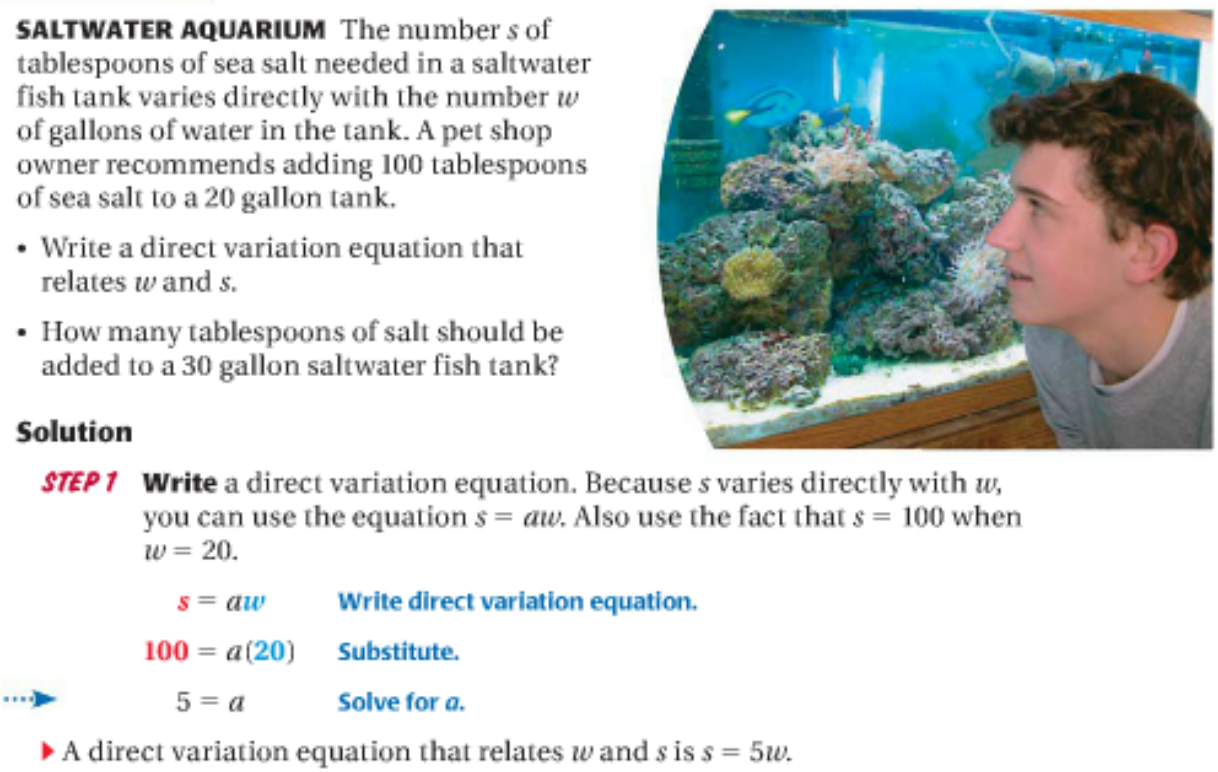
2. Draw a linear model paying attention to the closeness of the data points on either side of the line.

<table>
<tr><th>ath Scores</th></tr>
<tr><td>65</td></tr>
<tr><td>50</td></tr>
<tr><td>95</td></tr>
<tr><td>85</td></tr>
<tr><td>80</td></tr>
<tr><td>34</td></tr>
<tr><td>70</td></tr>
<tr><td>56</td></tr>
<tr><td>100</td></tr>
<tr><td>24</td></tr>
<tr><td>45</td></tr>
<tr><td>71</td></tr>
<tr><td>30</td></tr>
<tr><td>95</td></tr>
<tr><td>55</td></tr>
<tr><td>42</td></tr>
<tr><td>90</td></tr>
<tr><td>92</td></tr>
<tr><td>60</td></tr>
<tr><td>50</td></tr>
<tr><td>10</td></tr>
<tr><td>80</td></tr>
</table>
</td>
</tr>
<tr>
<td>8.5(E)   Solve problems involving direct variation;</td>
<td>
Students understand that two variables show direct variation provided y=kx and k is not equal to zero. k is the constant of proportionality or constant of variation. Student understands that the equation is read as y varies directly as x. Direct variation problems represent proportional relationships. Students also need to know that k=y/x.

Common Misconceptions.

Students not understanding that both k and m represent rate of change or slope.
</td>
</tr>
</table>

Examples:



SALTWATER AQUARIUM The number s of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w of gallons of water in the tank. A pet shop owner recommends adding 100 tablespoons of sea salt to a 20 gallon tank.

- Write a direct variation equation that relates w and s.
- How many tablespoons of salt should be added to a 30 gallon saltwater fish tank?

Solution

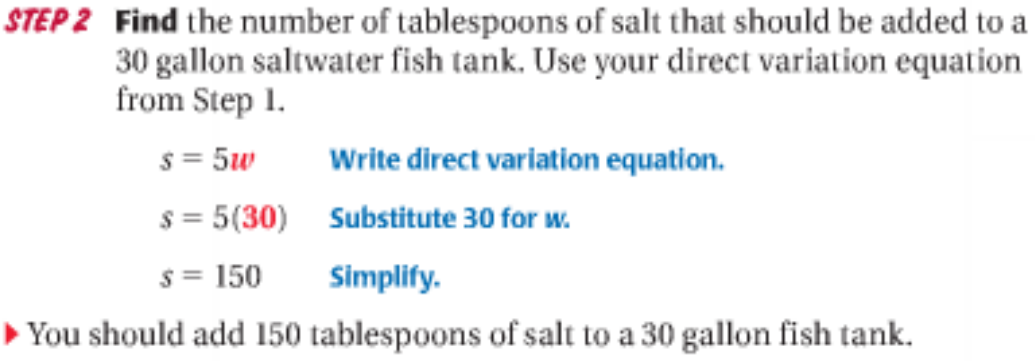
STEP 1 **Write** a direct variation equation. Because s varies directly with w, you can use the equation $s = aw$. Also use the fact that $s = 100$ when $w = 20$.

$$s = aw \quad \text{Write direct variation equation.}$$

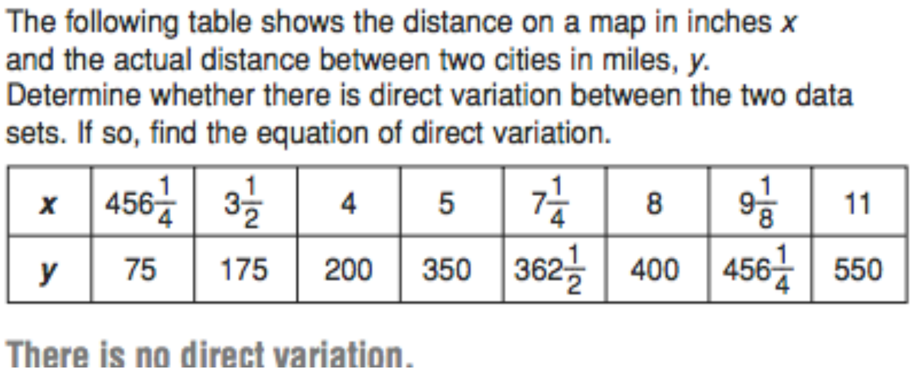
$$100 = a(20) \quad \text{Substitute.}$$

$$5 = a \quad \text{Solve for } a.$$

▸ A direct variation equation that relates w and s is $s = 5w$.

	 **STEP 2** **Find** the number of tablespoons of salt that should be added to a 30 gallon saltwater fish tank. Use your direct variation equation from Step 1. $s = 5w$ **Write direct variation equation.** $s = 5(30)$ **Substitute 30 for *w*.** $s = 150$ **Simplify.** ▶ You should add 150 tablespoons of salt to a 30 gallon fish tank.
8.5(F) Distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form *y* = *kx* or *y* = *mx* + *b*, where $b \neq 0$;	Student understands that linear situations can be represented four ways: in words, tables, graphs and as an equation. Student understands that non-proportional situations will affect the graph such that the y-intercept will no longer be zero; it will now be the number added or subtracted (b). Student understands that non-proportional situations affect the equation such that there will be a number added or subtracted (b). Student understand that the ratios represented in the table will no longer be equivalent (not proportional). Students understand from 7th grade that k=y/x is the constant of proportionality. We are now also referring to k as slope, rate of change, and unit rate. Proportional linear relationships will pass through the origin whereas non-proportional relationships will not. **Common Misconceptions.** Students not understanding that both k and m represent rate of change or slope. Students calculate slope (k or m) as x/y instead of y/x.

Examples:

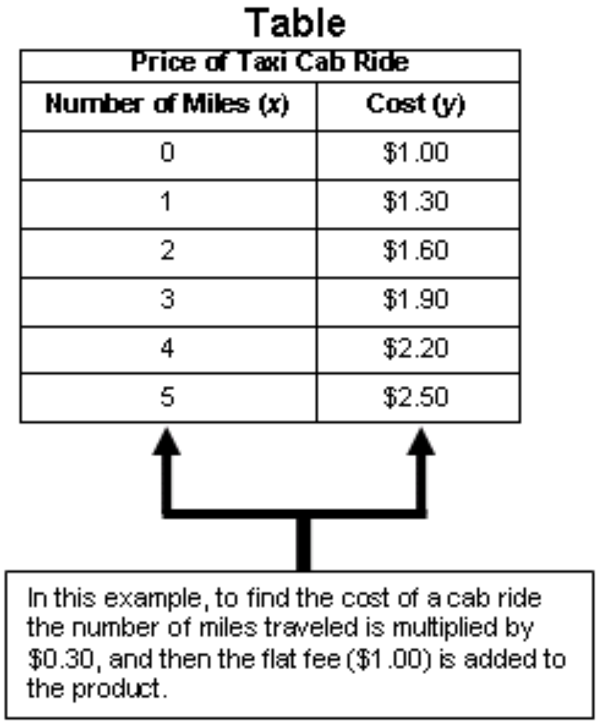


The following table shows the distance on a map in inches *x* and the actual distance between two cities in miles, *y*. Determine whether there is direct variation between the two data sets. If so, find the equation of direct variation.

x	$456\frac{1}{4}$	$3\frac{1}{2}$	4	5	$7\frac{1}{4}$	8	$9\frac{1}{8}$	11
y	75	175	200	350	$362\frac{1}{2}$	400	$456\frac{1}{4}$	550

There is no direct variation.

Example – N
on-proportional linear relationships (y = mx + b, where b is not 0):
Ajax Taxicab Company charges a flat fee of $1.00 plus $0.30 per mile to ride in a cab. (Assumption: The flat fee is incurred as soon as you enter the cab.)
Rule in words: To determine the cost of an Ajax Taxicab ride, multiply the number of miles traveled by $0.30, and then add $1.00 (the flat fee) to the product.
Rule in Equation: If y represents the total cost of an Ajax Taxicab ride of x miles, then the relationship can be expressed as an equation in the form of y = mx + b, where m represents the cost per mile ($0.30/mile) and b represents the flat fee ($1.00).
Total Cost = Cost per Mile • Number of Miles + Flat Fee
$y = 0.30 \cdot x + 1.00$, or $y = 0.30x + 1.00$

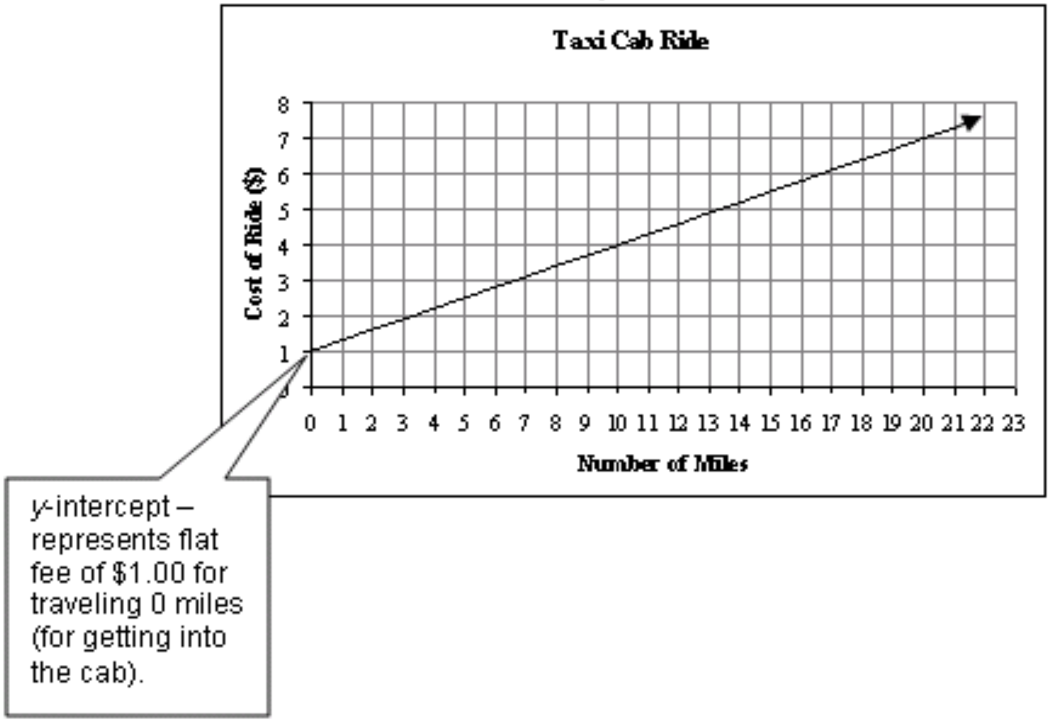


Table

Price of Taxi Cab Ride	
Number of Miles (*x*)	**Cost (*y*)**
0	$1.00
1	$1.30
2	$1.60
3	$1.90
4	$2.20
5	$2.50

In this example, to find the cost of a cab ride the number of miles traveled is multiplied by $0.30, and then the flat fee ($1.00) is added to the product.

Graph



y-intercept – represents flat fee of $1.00 for traveling 0 miles (for getting into the cab).

Example – Proportional linear relationships (y = kx):

A farmer sells apples for $2.00 per pound.
Rule in words: If apples cost $2.00 per pound, then the total cost of purchasing apples is $2.00 multiplied by the number of pounds purchased.
Rule in equation: If the price of apples per pound (k) is $2.00, the total cost (y) of purchasing apples at $2.00 per pound is calculated by multiplying the number of pounds of apples (x) purchased by $2.00. Therefore, the equation in y = kx form that represents the relationship between the total cost of the apples and the number of pounds of apples is y = 2x.

<table>
<tr>
<td></td>
<td>
Apples for Sale
$2.00 per pound

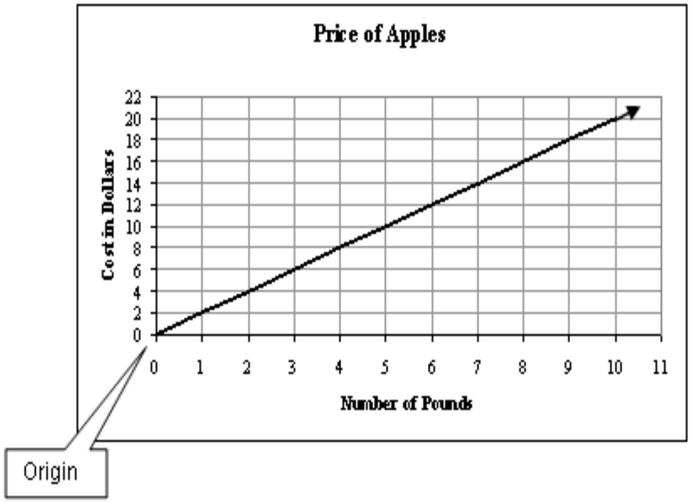
Graph


Table
<table>
<tr><th colspan="2">Price of Apples</th></tr>
<tr><th>Number of Pounds of Apples (<i>x</i>)</th><th>Cost (<i>y</i>)</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>$2.00</td></tr>
<tr><td>2</td><td>$4.00</td></tr>
<tr><td>3</td><td>$6.00</td></tr>
<tr><td>4</td><td>$8.00</td></tr>
<tr><td>5</td><td>$10.00</td></tr>
</table>

In this example, to find the cost, the number of pounds is always multiplied by $2.00 which is the constant rate of change (<i>k</i>).
</td>
</tr>
<tr>
<td>8.5(G) Identify functions using sets of ordered pairs, tables, mappings, and graphs;</td>
<td>Student understands the different function vocabulary: relation, function, domain, range, input, output and vertical line test. Student can determine whether a relation is a function by using ordered pairs, tables, maps, and graphs.

Common Error:

1. Students have trouble understanding that in a function, x’s cannot repeat, but y’s can.</td>
</tr>
</table>

Examples:

From an Ordered Pair:

Function: {(2,1)(3,4)(6,7)}

Not a Function: {(2,1)(2,2)(2,3)}

From a Table:

Function:

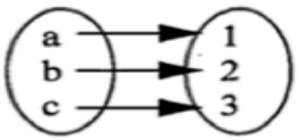
x	y
2	4
4	8
6	12

Not a Function:

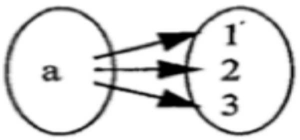
x	y
4	-2
1	-1
0	0
1	1

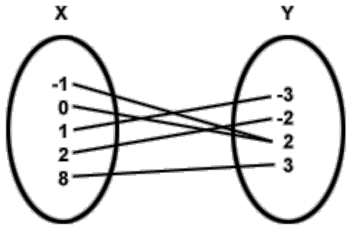
From a Map:

Function:



Not a Function:





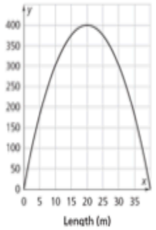
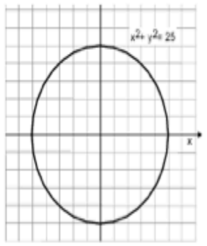
Determining whether a relation is a function:
From a table: If any x-value is paired with only one y-value it is a function.

X	-1	0	1	2
Y	-1	3	7	11

This is a function. Each domain or x-value is paired with only one range or y-value.

X	0	-1	-2	0
Y	2	4	6	8

This is not a function. The domain value, or x value of 0 repeats.

	Graphs Students recognize graphs such as the one below is a function using the vertical line test, showing that each *x*-value has only one *y*-value;  whereas, graphs such as the following are not functions since there are 2 *y*-values for multiple *x*-value. 
8.5(H) Identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems;	Student must be able to identify proportional and non-proportional relationships. Student understands that proportional functions will be in the form y=kx. Student understands that non-proportional functions will be in the form y=mx+b. **Common Misconceptions.** Students forget to add the constant (ie. base charge, initial fee, starting amount) **Examples:** Non-Proportional relationships in the real world are those that have an initial feel. For example, a gym with an enrollment fee of $35.00 and cost 20.00.

	Identify the two situations as proportional or non-proportional: While Miguel drinks his coffee, he decides to compare some text plans to see which one would work better for him. His cell phone company offers two plans. The basic plan charges 7 cents per text message sent or received. The advanced plan lets Miguel send text, pictures and video, and it only charges 2 cents per message. Score! Oh, but wait—the advanced plan has a $10 monthly fee. So how is Miguel going to decide which plan is actually the better deal for him? Basic plan: $y=.07x$ (proportional) Advanced plan: $y=.02x + 10$ (non-proportional)
8.5(I) Write an equation in the form *y* = *mx* + *b* to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.	Given two quantities, student must be able to determine which quantity is the independent and dependent variables. Student must determine the slope (m) and y-intercept (b) from a verbal, numerical, table and graphical representation in order to write the equation in y=mx+b. **Common Misconceptions.** 1. Students reverse m and b when writing the equation (write y=bx +m) 2. Calculating slope as x/y instead of y/x.

Examples:

Example 1:

1. Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

Solution:

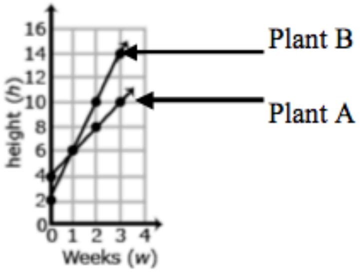
Let W = number of weeks
Let H = height of the plant after W weeks

Plant A		
W	H	
0	4	(0, 4)
1	6	(1, 6)
2	8	(2, 8)
3	10	(3, 10)

Plant B		
W	H	
0	2	(0, 2)
1	6	(1, 6)
2	10	(2, 10)
3	14	(3, 14)

2. Based on the coordinates from the table, graph lines to represent each plant.

Solution:

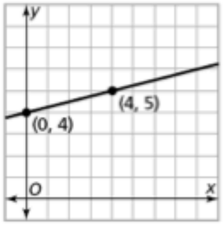


3. Write an equation that represents the growth rate of Plant A and Plant B.

Solution: Plant A $H = 2W + 4$
Plant B $H = 4W + 2$

Example 2:

Write an equation that models the linear relationship in the graph below.



Solution: The y-intercept is 4. The slope is ¼ , found by moving up 1 and right 4 going from (0, 4) to (4, 5). The linear equation would be $y = \frac{1}{4}x + 4$.

Equations:

In a linear equation the coefficient of x is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Point and Slope:

Students write equations to model lines that pass through a given point with the given slope.

Example 2:

A line has a zero slope and passes through the point (-5, 4). What is the equation of the line?

Solution: $y = 4$

Example 3:

Write an equation for the line that has a slope of ½ and passes though the point (-2, 5)

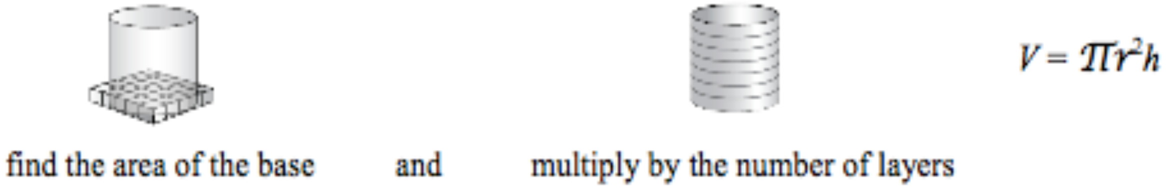
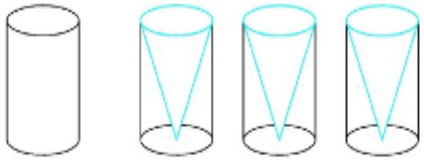
Solution: $y = \frac{1}{2}x + 6$

Students could multiply the slope ½ by the x-coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. **Note that point-slope form is not an expectation at this level.** Students use the slope and y-intercepts to write a linear function in the form $y = mx + b$.

Contextual Situations:

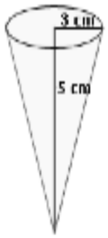
In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be "converted" to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

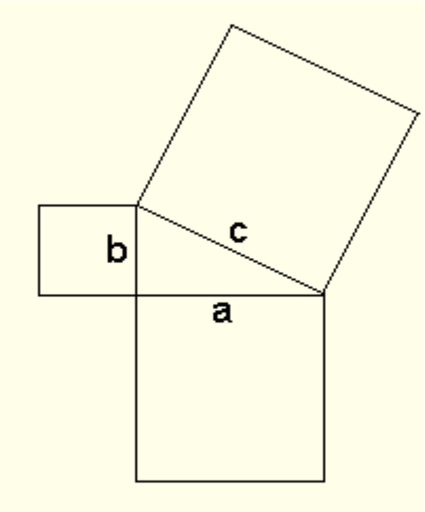
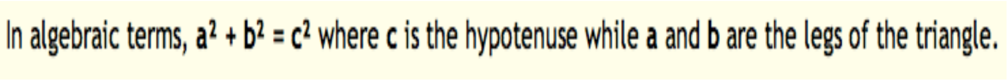
Expressions, Equations, and Relationships: **TEK 8.6**	**The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:**
8.6(A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height 8.6(B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas	“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for *all* students.,Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, finding the area of the base ∏r2 and multiplying by the number of layers (the height).  Students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is 1/3 the volume of a cylinder having the same base area and height.  $V = \frac{1}{3}\pi r^2 h$ or $V = \underline{\pi r^2 h}$ **Common Misconceptions**:

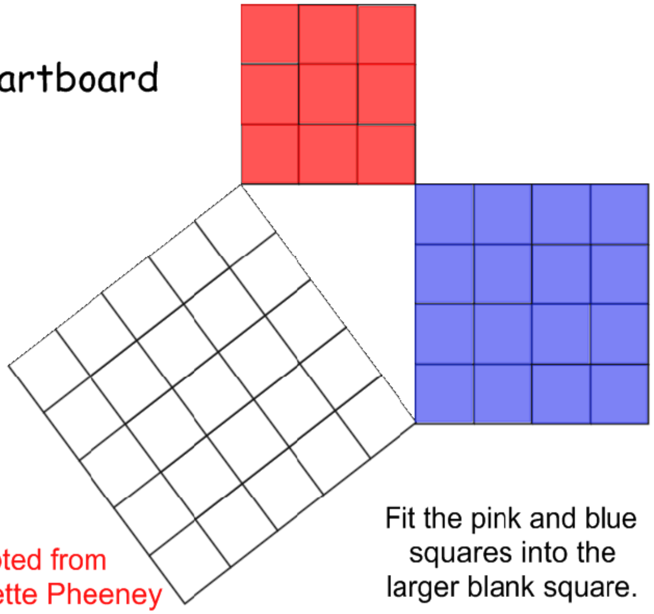
1. Student thinks B and b are interchangeable. B stands for area of the base shape where b is used as a linear measurement.
2. Students calculate r squared as r times 2. (r squared is not the same as the diameter)
3. Students try to calculate the cone formula using a decimal (.3) for ⅓.
4. Students have difficulty trying to reduce in the problem (on cones). For example: simplifying before squaring when the radius in a multiple of 3, or trying to calculate the entire formula THEN dividing by three as the last part of the formula when very large numbers are involved.

Examples:

Example 1: James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.

	Solution: $V = \pi r^2 h$ $V = 3.14\ (40)^2(100)$ $V = 502{,}400\text{ cm}^3$ The answer could also be given in terms of π: $V = 160{,}000\ \pi$ Example 2: How much yogurt is needed to fill the cone to the right? Express your answers in terms of Pi.  *Solution:* $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi(3^2)(5)$ $V = \frac{1}{3}\pi\ (45)$ $V = 15\ \pi\text{ cm}^3$
8.6(C) use models and diagrams to explain the Pythagorean theorem	Students know that c, the hypotenuse, is always the longest side of the right triangle. The legs are denoted by a and b, using the formula $a^2+b^2=c^2$. Students should be introduced to and familiar with Pythagorean Triples including (3,4,5) and (5,12,13). Students should also be able to use graph paper to show the Pythagorean Theorem. Given any variable used on a triangle, students should be able to write the Pythagorean theorem using the indicated variables. **Common Misconceptions:** 1. Students substitute measurement for leg as measure for hypotenuse. 2. Understanding that the area formed from each side is a square.

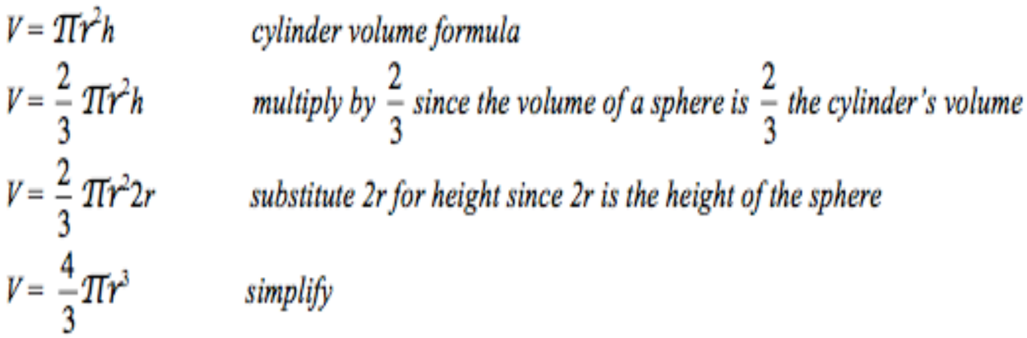
	Examples:   In algebraic terms, **a^2** + **b^2** = **c^2** where **c** is the hypotenuse while **a** and **b** are the legs of the triangle.

	Smartboard  Adapted from Pierette Pheeney Fit the pink and blue squares into the larger blank square.
Expressions, Equations and Relationships: TEK 8.7	**The student applies mathematical process standards to use geometry to solve problems. The student is expected to:**
8.7(A) solve problems involving the volume of cylinders, cones, and spheres	Students should be familiar with 3-dimensional figures including cylinders, cones, and spheres and identify the attributes of the figure including Base, height, and/or radius. "Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of **why** the formula works and how the formula relates to the measure (volume) and the figure.

Common Misconceptions:

1. Students may use diameter rather than radius in calculations.

A sphere can be enclosed with a cylinder, which has the same radius and height of the sphere (Note: the height of the cylinder is twice the radius of the sphere). If the sphere is flattened, it will fill 2/3 of the cylinder. Based on this model, students understand that the volume of a sphere is 2/3 the volume of a cylinder with the same radius and 2/3 height. The height of the cylinder is the same as the diameter of the sphere or 2*r*. Using this information, the formula for the volume of the sphere can be derived in the following way:

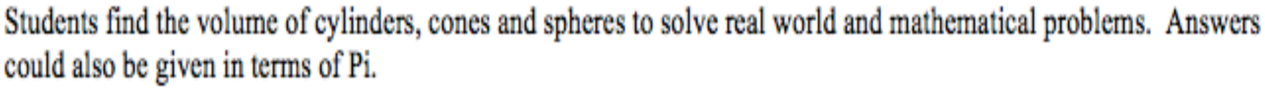


$V = \pi r^2 h$ *cylinder volume formula*

$V = \frac{2}{3}\pi r^2 h$ *multiply by* $\frac{2}{3}$ *since the volume of a sphere is* $\frac{2}{3}$ *the cylinder's volume*

$V = \frac{2}{3}\pi r^2 2r$ *substitute 2r for height since 2r is the height of the sphere*

$V = \frac{4}{3}\pi r^3$ *simplify*



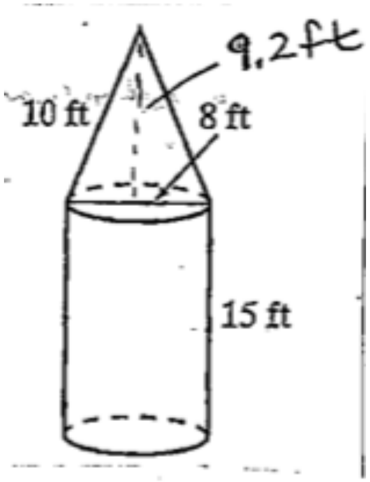
Students find the volume of cylinders, cones and spheres to solve real world and mathematical problems. Answers could also be given in terms of Pi.

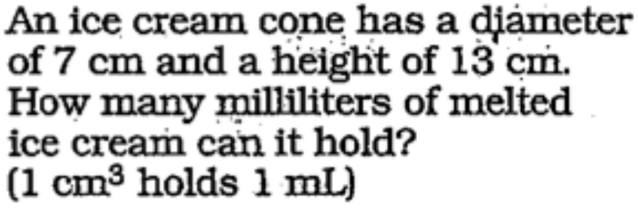
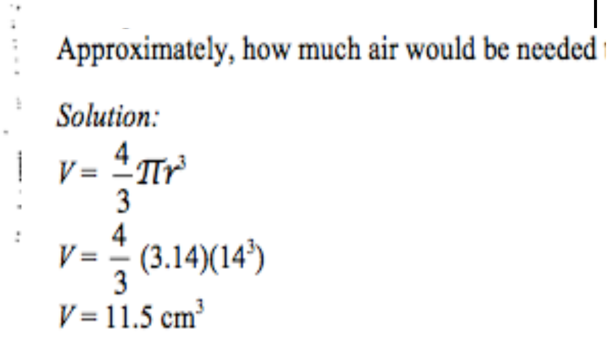
Examples:

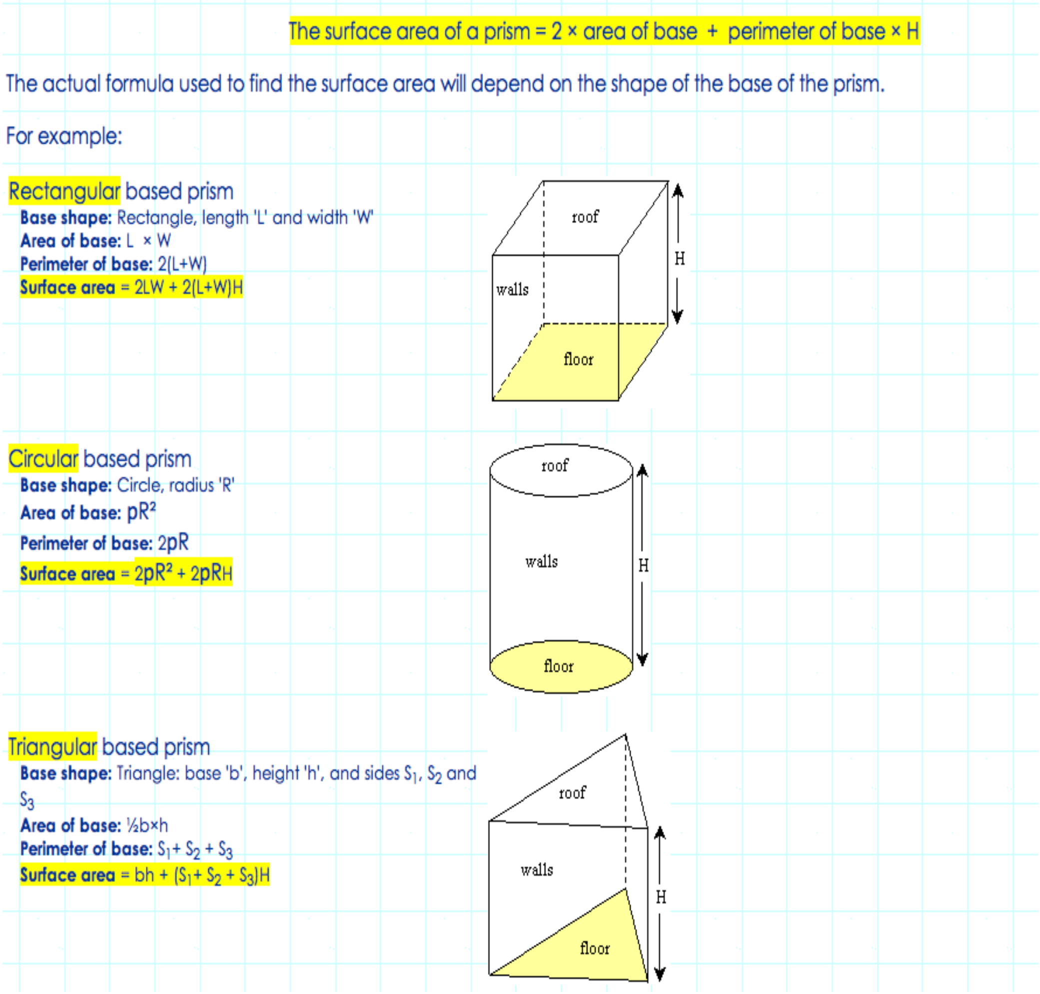
How much space is in a cylinder if three tennis balls with diameter of 3 inches are packed into the can. Assume the tennis balls touch the sides, top and bottom of cylinder.

A peanut butter jar has a height of 5.9 in. and diameter of 3.6 in. One cubic inch holds 0.45 oz of peanut butter. How many ounces will fit in the jar?

Jo was comparing two cylinders that both had a radius of 5 cm. The first had a height of 10 cm, and the other a height of 20 cm. How many times greater was the volume of the larger cylinder?



	 An ice cream cone has a diameter of 7 cm and a height of 13 cm. How many milliliters of melted ice cream can it hold? (1 cm^3 holds 1 mL)  Approximately, how much air would be needed *Solution:* $V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(3.14)(14^3)$ $V = 11.5\ \text{cm}^3$
8.7(B) Use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders	Students must have a strong foundation in identifying the net of a shape. Students must be able to make the connection between finding the area of each "part" of the shape then adding them together leads to using the formula involving P (perimeter of the base shape). S = 2B +Ph Students must be able to substitute the correct area formula for B (area of base figure). Students must be able to recognize key words in the problem that refer to ONLY the lateral surface area of an object. **Common Misconceptions:** 1. Students confuse B=area of base figure and b=linear length. Same for P=perimeter of base figure. (This is NOT included on the STAAR reference chart) 2. Students have trouble identifying which shape is the base figure. They assume that what is the bottom figure is the base shape. Must reinforce that on a triangular prism, the triangle is the base; on a cylinder, the circle is the base shape. 3. Students assume they are always finding the total surface area and not just the lateral surface area at times. **Examples:**



The surface area of a prism = 2 × area of base + perimeter of base × H

The actual formula used to find the surface area will depend on the shape of the base of the prism.

For example:

Rectangular based prism

Base shape: Rectangle, length 'L' and width 'W'
Area of base: L × W
Perimeter of base: 2(L+W)
Surface area = 2LW + 2(L+W)H

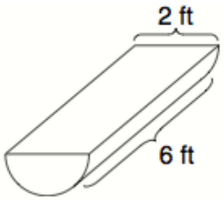
Circular based prism

Base shape: Circle, radius 'R'
Area of base: pR^2
Perimeter of base: $2pR$
Surface area = $2pR^2 + 2pRH$

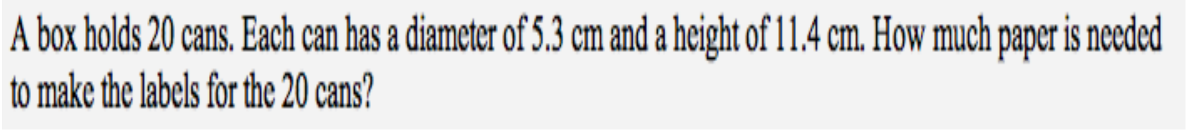
Triangular based prism

Base shape: Triangle: base 'b', height 'h', and sides S_1, S_2 and S_3
Area of base: ½b×h
Perimeter of base: $S_1 + S_2 + S_3$
Surface area = $bh + (S_1 + S_2 + S_3)H$

How much metal is required to make the trough pictured below?

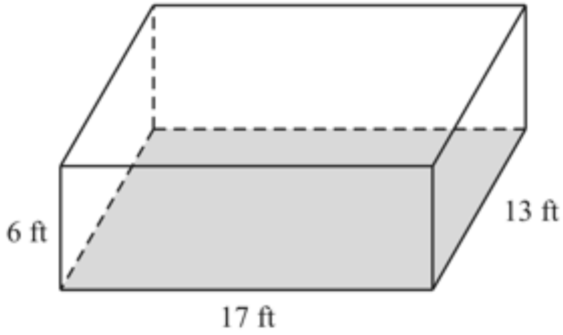


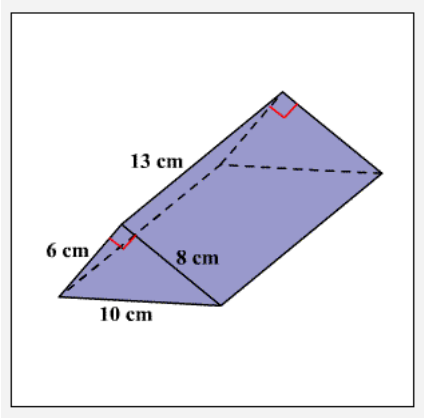
A 22.0 ft^2 **C** 44.0 ft^2
B 34.0 ft^2 **D** 56.7 ft^2



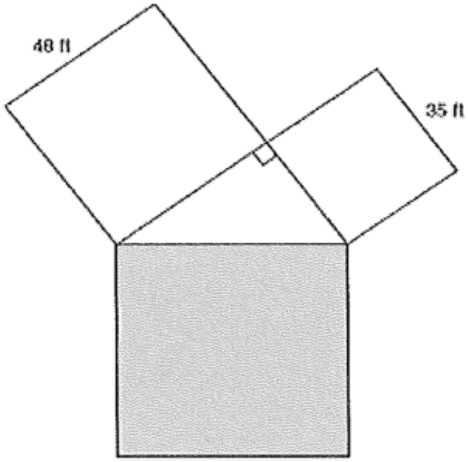
A box holds 20 cans. Each can has a diameter of 5.3 cm and a height of 11.4 cm. How much paper is needed to make the labels for the 20 cans?

The room below has the given dimensions. All surfaces except the floor are to be painted. If a gallon of paint will cover 60 cu. ft, how many gallons of paint will you need to purchase to complete the job?



	Boy scouts are preparing for a campout. They want to make a tent with the following dimensions. How much material would be needed to create the tent, which includes the sides, the floor, and the flaps for the front and back? 
8.7(C) Use the Pythagorean Theorem and its converse to solve problems	Students can explain the Pythagorean Theorem, understanding that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle. **Common Misconceptions.** 1. Students not identifying the longest dimension as the hypotenuse when given three measurements. 2. Students forget to find the square root of the final measurement. 3. Students do the leg times 2 and not squared. **Examples:**

The drawing below shows three squares joined at their vertices to form a right triangle.



What is the area of the shaded square?

A) 3529 ft^2

B) 1079 ft^2

C) 6889 ft^2

D) 169 ft^2

Example 1:

The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not?

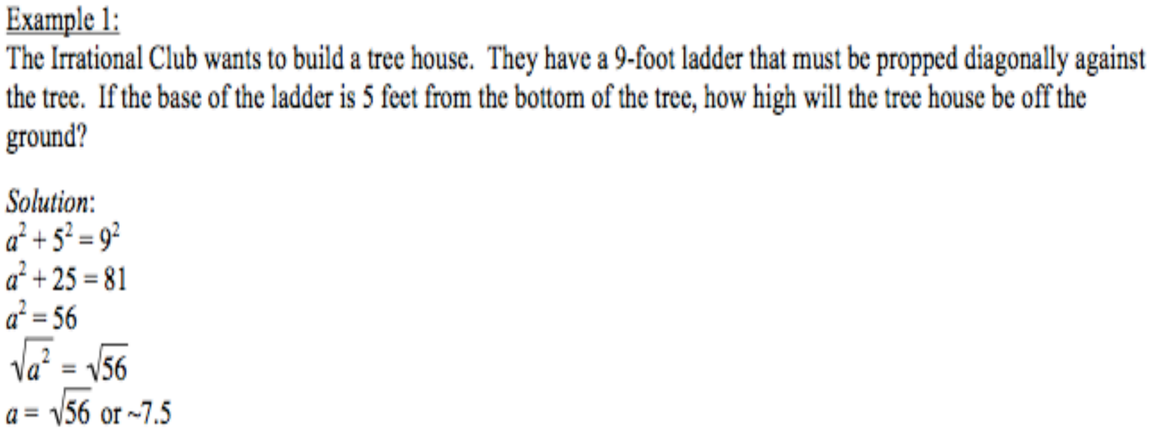
Solution: If these three towns form a right triangle, then 300 would be the hypotenuse since it is the greatest distance.

$180^2 + 240^2 = 300^2$

$32400 + 57600 = 90000$

$90000 = 90000$ ✓

These three towns form a right triangle.



Example 1:

The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?

Solution:

$a^2 + 5^2 = 9^2$

$a^2 + 25 = 81$

$a^2 = 56$

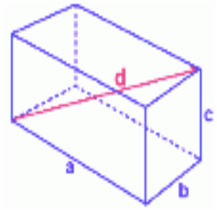
$\sqrt{a^2} = \sqrt{56}$

$a = \sqrt{56}$ or ~7.5

Example 2:

Find the length of *d* in the figure to the right if *a* = 8 in., *b* = 3 in. and *c* = 4in.

Solution:



First find the distance of the hypotenuse of the triangle formed with legs *a* and *b*.

$8^2 + 3^2 = c^2$

$64^2 + 9^2 = c^2$

$73 = c^2$

$\sqrt{73} = \sqrt{c^2}$

$\sqrt{73}$ in. $= c$

The $\sqrt{73}$ is the length of the base of a triangle with *c* as the other leg and *d* is the hypotenuse. To find the length of *d*:

$\sqrt{73}^2 + 4^2 = d^2$

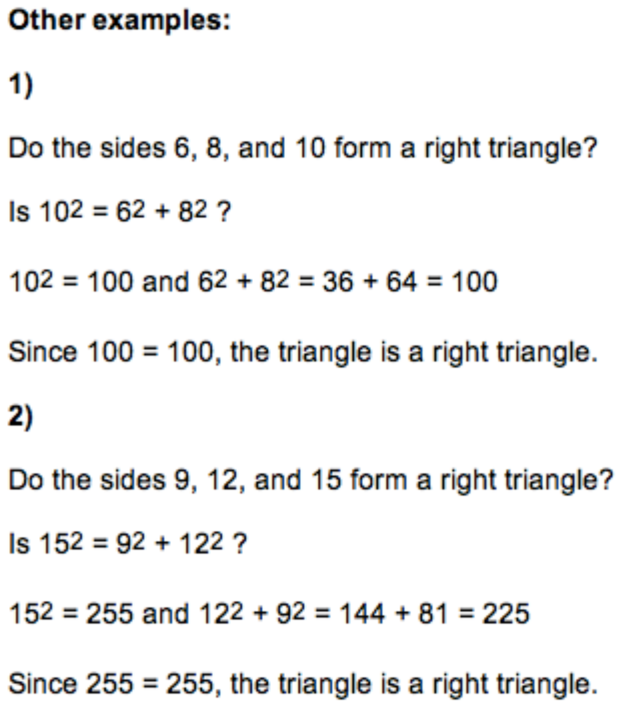
$73 + 16 = d^2$

$89 = d^2$

$\sqrt{89} = \sqrt{d^2}$

$\sqrt{89}$ in. $= d$

Based on this work, students could then find the volume or surface area.

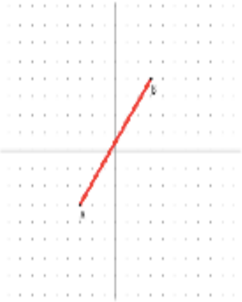
	 **Other examples:** **1)** Do the sides 6, 8, and 10 form a right triangle? Is $10^2 = 6^2 + 8^2$? $10^2 = 100$ and $6^2 + 8^2 = 36 + 64 = 100$ Since 100 = 100, the triangle is a right triangle. **2)** Do the sides 9, 12, and 15 form a right triangle? Is $15^2 = 9^2 + 12^2$? $15^2 = 255$ and $12^2 + 9^2 = 144 + 81 = 225$ Since 255 = 255, the triangle is a right triangle.
8.7(D) Determine the distance between two points on a coordinate plane using the Pythagorean Theorem	Students must have a solid knowledge of points on a coordinate plane. Student will need to draw the right triangle that is formed from two given points. They can then count the number of units for the two legs then use the Pythagorean Theorem to find the hypotenuse. Without drawing the right triangle formed by two points, students can find the side lengths of the legs by calculating the differences in both x-coordinates then y-coordinates and taking their absolute values. This will give the leg lengths for use in Pythagorean theorem. **Common Misconceptions:** 1. Students do not pay attention to the values on the axis (ie. could be counting by two's, five's, etc) 2. Students forget to find the square root of the found measurement. 3. Students think a diagonal unit on grid paper is the same value as one horizontal or vertical unit.

Examples:

NOTE: The use of the distance formula is not an expectation.

Example 1:

Find the length of $\overline{AB}$.



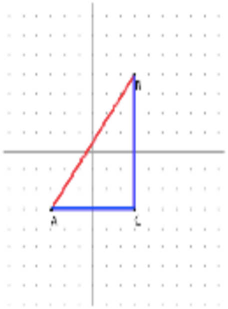
Solution:

1. Form a right triangle so that the given line segment is the hypotenuse.
2. Use Pythagorean Theorem to find the distance (length) between the two points.

$$6^2 + 7^2 = c^2$$

$$36 + 49 = c^2$$

$$85 = c^2$$



Example 2:

Find the distance between (-2, 4) and (-5, -6).

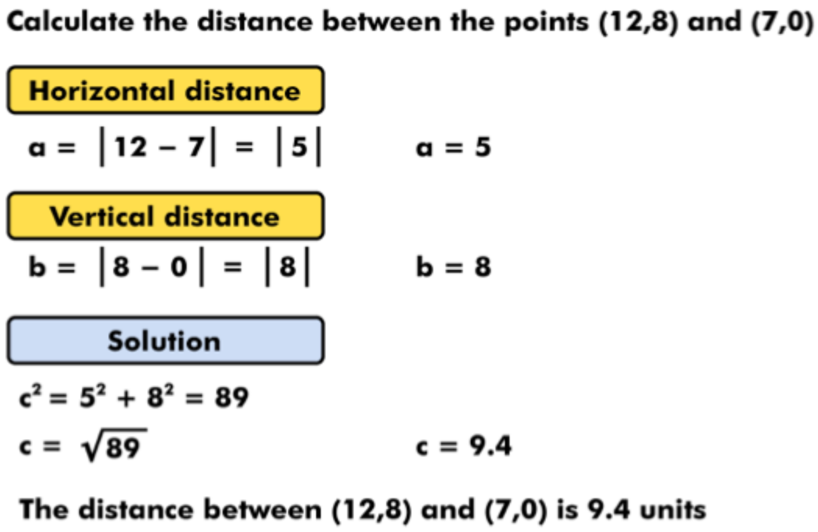
Solution:

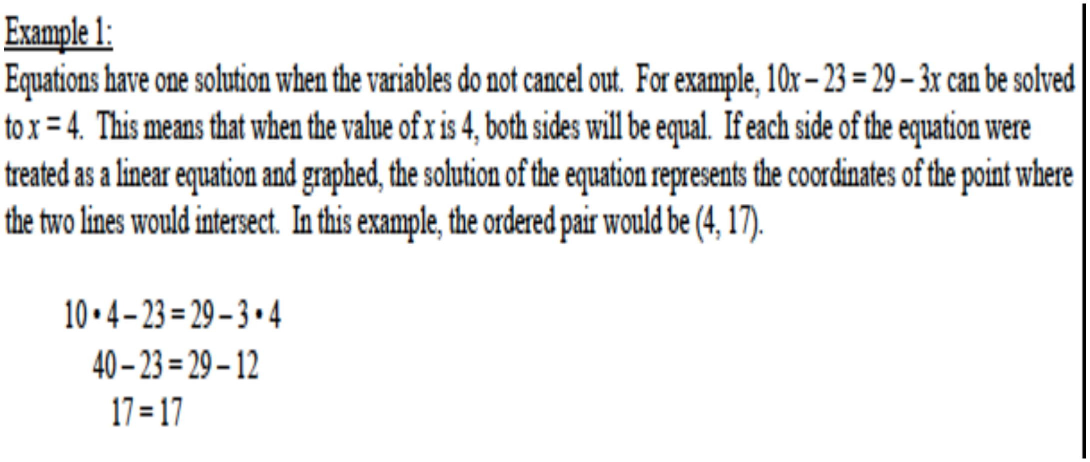
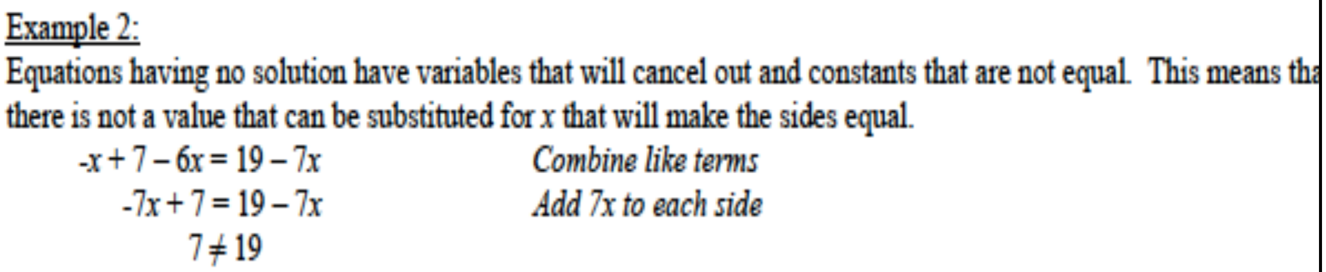
The distance between -2 and -5 is the horizontal length; the distance between 4 and -6 is the vertical distance.

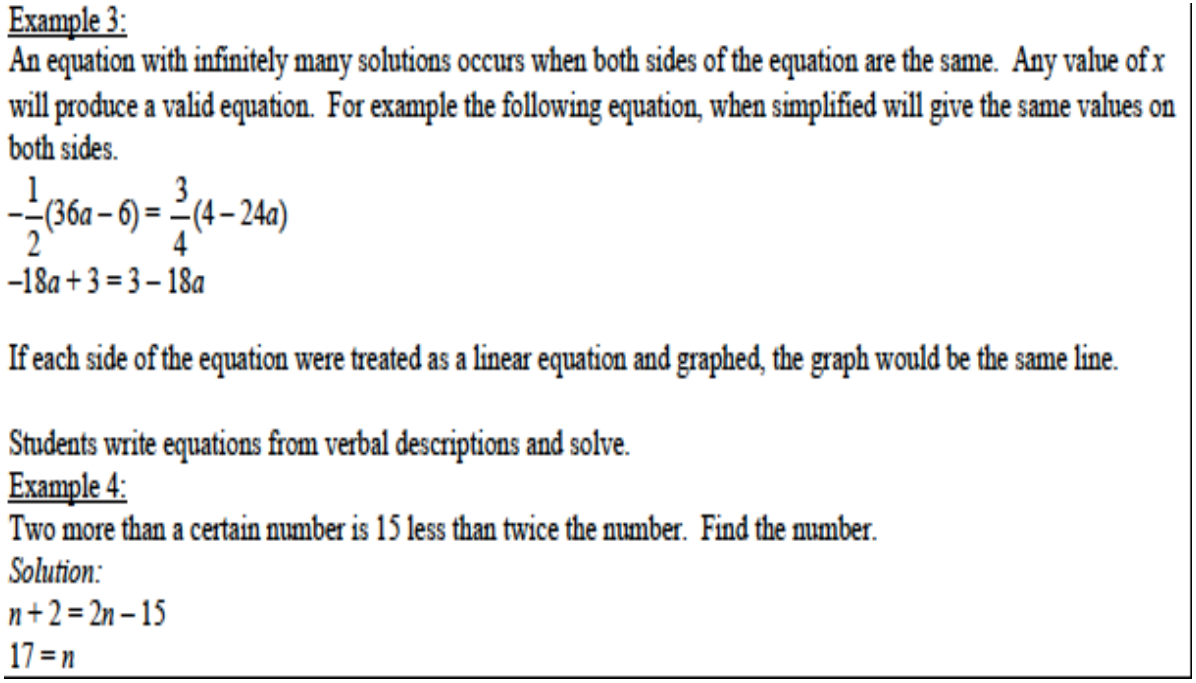
Horizontal length: 3

Vertical length: 10

$$10^2 + 3^2 = c^2$$
$$100 + 9 = c^2$$
$$109 = c^2$$
$$\sqrt{109} = \sqrt{c^2}$$
$$\sqrt{109} = c$$

	 Calculate the distance between the points (12,8) and (7,0) **Horizontal distance** $a = \|12 - 7\| = \|5\|$ $a = 5$ **Vertical distance** $b = \|8 - 0\| = \|8\|$ $b = 8$ **Solution** $c^2 = 5^2 + 8^2 = 89$ $c = \sqrt{89}$ $c = 9.4$ The distance between (12,8) and (7,0) is 9.4 units
Expressions, Equations and Relationships: TEK 8.8	**The student applies mathematical process standards to use one variable equations or inequalities in problem situations. The student is expected to:**
8.8(A) Write one---variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants 8.8(B) Write a corresponding real---world problem when given a one---variable equation	Students recognize that the solution to the equation is the value(s) of the variable, which makes it a true equality when substituted back into the equation. A one variable equation can have one solution, no solution, or infinitely many solutions. Students must have knowledge/understanding of WHY and WHEN you would reverse the inequality symbol when solving inequalities. Students must have understanding of combining like terms (ex: 4a - 7a) and use of the distributive property. **Misconceptions:** 1. Student forgets to reverse the inequality symbol when multiplying/dividing both side of equation by a negative number. 2. Students does not use property of equality correctly (for example: 7 - 6x = 12, student will add 7 instead of subtract 7).

or inequality with variables on both sides of the equal sign using rational number coefficients and constants 8.8(C) Model and solve one---variable equations with variables on both sides of the equal sign that represent mathematical and real---world problems using rational number coefficients and constants	3. Students move all constants and variables to one side of the equation. Examples:  Example 1: Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17). $10 \cdot 4 - 23 = 29 - 3 \cdot 4$ $40 - 23 = 29 - 12$ $17 = 17$  Example 2: Equations having no solution have variables that will cancel out and constants that are not equal. This means tha there is not a value that can be substituted for x that will make the sides equal. $-x + 7 - 6x = 19 - 7x$ *Combine like terms* $-7x + 7 = 19 - 7x$ *Add 7x to each side* $7 \neq 19$



Example 3:

An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.

$$-\frac{1}{2}(36a-6)=\frac{3}{4}(4-24a)$$

$$-18a+3=3-18a$$

If each side of the equation were treated as a linear equation and graphed, the graph would be the same line.

Students write equations from verbal descriptions and solve.

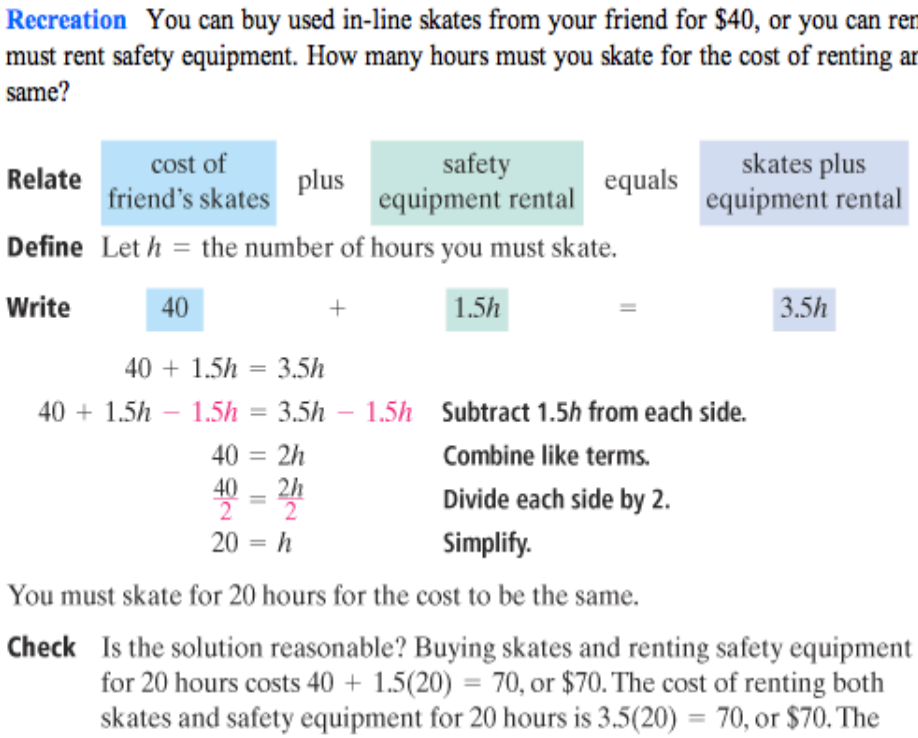
Example 4:

Two more than a certain number is 15 less than twice the number. Find the number.

Solution:

$$n+2=2n-15$$

$$17=n$$



Recreation You can buy used in-line skates from your friend for \$40, or you can rent some. Either way, you must rent safety equipment. How many hours must you skate for the cost of renting and buying skates to be the same?

Relate	cost of friend's skates	plus	safety equipment rental	equals	skates plus equipment rental
Define	Let h = the number of hours you must skate.				
Write	40	+	$1.5h$	=	$3.5h$

$$40 + 1.5h = 3.5h$$

$$40 + 1.5h - 1.5h = 3.5h - 1.5h$$ **Subtract 1.5*h* from each side.**

$$40 = 2h$$ **Combine like terms.**

$$\frac{40}{2} = \frac{2h}{2}$$ **Divide each side by 2.**

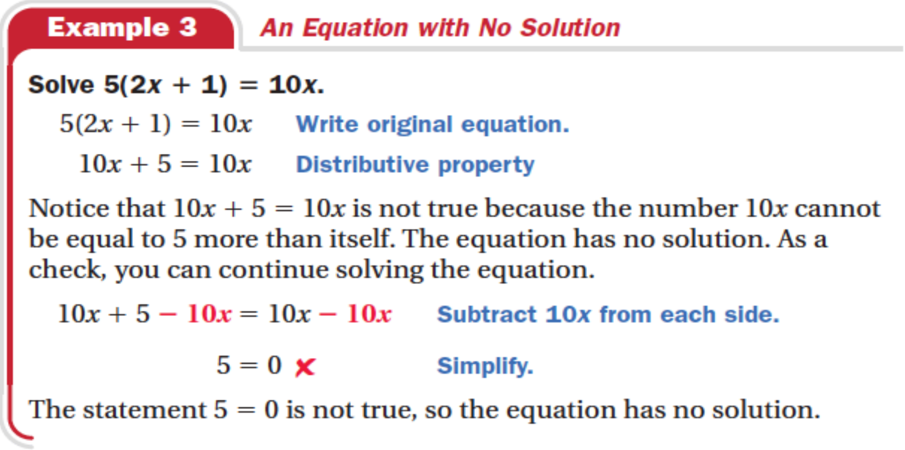
$$20 = h$$ **Simplify.**

You must skate for 20 hours for the cost to be the same.

Check Is the solution reasonable? Buying skates and renting safety equipment for 20 hours costs $40 + 1.5(20) = 70$, or \$70. The cost of renting both skates and safety equipment for 20 hours is $3.5(20) = 70$, or \$70. The

Sanjeev and Jack have the same amount of money. Jack buys 3 burgers and receives \$4.72 change. KJ buys 5 burgers and receives \$1.20 change.

Find the cost of a burger (*b*).



Example 3 *An Equation with No Solution*

Solve $5(2x + 1) = 10x$.

$5(2x + 1) = 10x$ Write original equation.

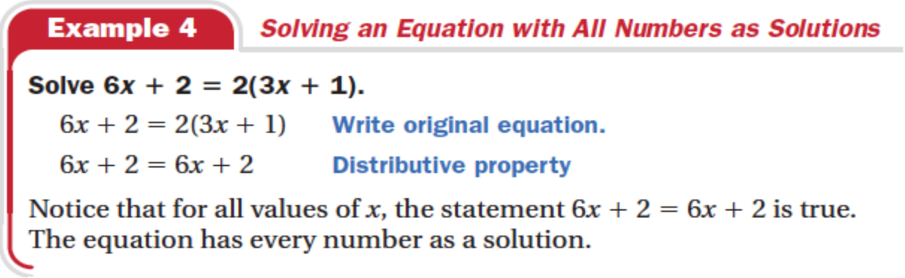
$10x + 5 = 10x$ Distributive property

Notice that $10x + 5 = 10x$ is not true because the number $10x$ cannot be equal to 5 more than itself. The equation has no solution. As a check, you can continue solving the equation.

$10x + 5 - 10x = 10x - 10x$ Subtract $10x$ from each side.

$5 = 0$ ✗ Simplify.

The statement $5 = 0$ is not true, so the equation has no solution.



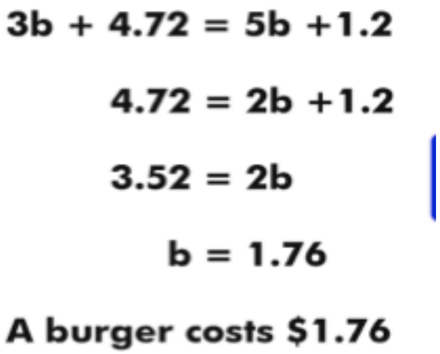
Example 4 *Solving an Equation with All Numbers as Solutions*

Solve $6x + 2 = 2(3x + 1)$.

$6x + 2 = 2(3x + 1)$ Write original equation.

$6x + 2 = 6x + 2$ Distributive property

Notice that for all values of x, the statement $6x + 2 = 6x + 2$ is true. The equation has every number as a solution.



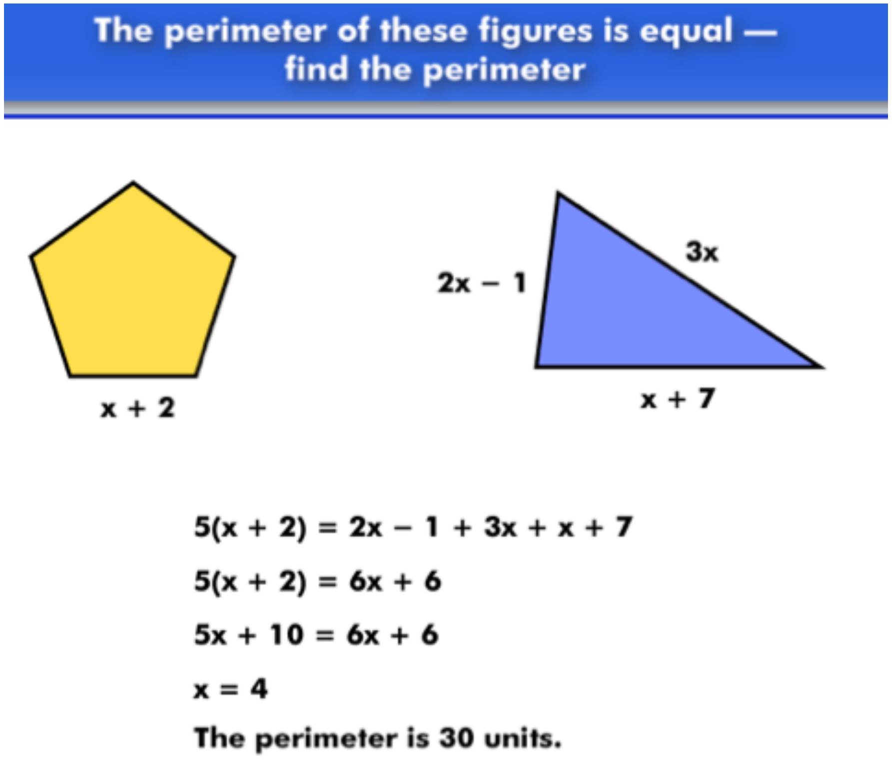
$3b + 4.72 = 5b + 1.2$

$4.72 = 2b + 1.2$

$3.52 = 2b$

$b = 1.76$

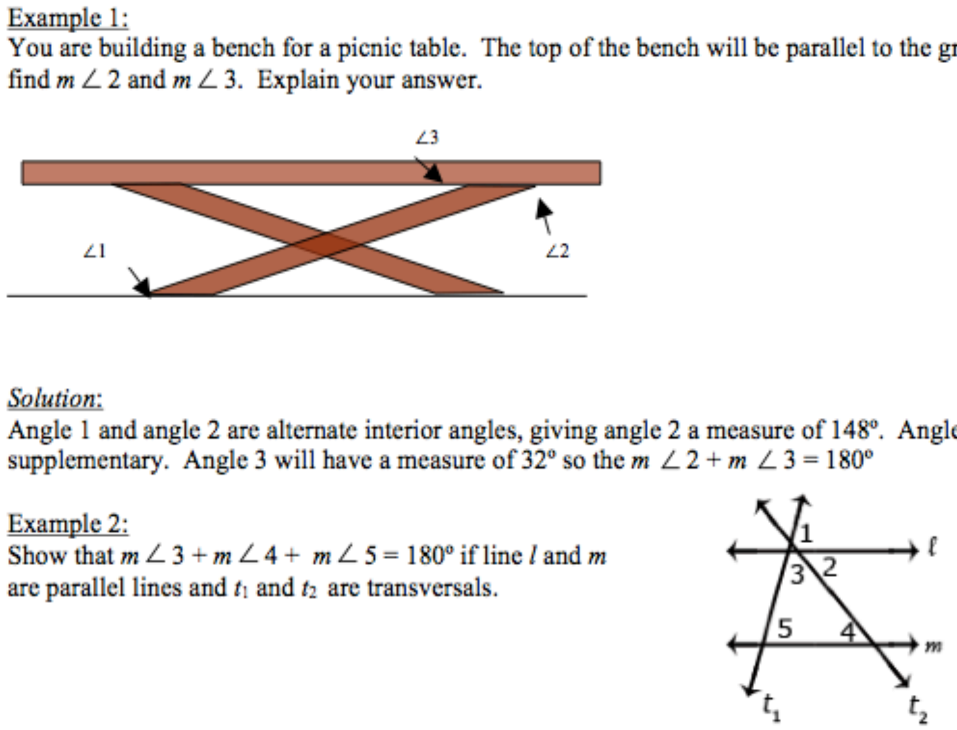
A burger costs $1.76

	 The perimeter of these figures is equal — find the perimeter $5(x + 2) = 2x - 1 + 3x + x + 7$ $5(x + 2) = 6x + 6$ $5x + 10 = 6x + 6$ $x = 4$ The perimeter is 30 units.
8.8(D) Use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-criterion for similarity of triangles	Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360 degrees). Using these relationships, students use deductive reasoning to find the measure of missing angles. Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Common Misconceptions.

Students do not have a foundational knowledge of key vocabulary such as vertical angles, alternate interior and exterior angles, etc, and their relationships.

Examples:



Example 1:

You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If $m\angle 1 = 148^\circ$, find $m\angle 2$ and $m\angle 3$. Explain your answer.

Solution:

Angle 1 and angle 2 are alternate interior angles, giving angle 2 a measure of 148°. Angle 2 and angle 3 are supplementary. Angle 3 will have a measure of 32° so the $m\angle 2 + m\angle 3 = 180^\circ$

Example 2:

Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$ if line l and m are parallel lines and t_1 and t_2 are transversals.

Solution: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\angle 5 \cong \angle 1$ corresponding angles are congruent therefore $\angle 1$ can be substituted for $\angle 5$

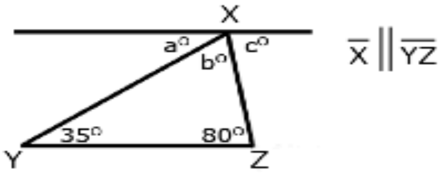
$\angle 4 \cong \angle 2$ alternate interior angles are congruent therefore $\angle 4$ can be substituted for $\angle 2$

Therefore $\angle 3 + \angle 4 + \angle 5 = 180^\circ$

Students can informally conclude that the sum of the angles in a triangle is 180° (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.

Example 3:

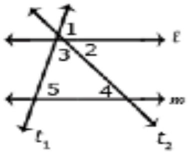
In the figure below Line X is parallel to Line $\overline{YZ}$. Prove that the sum of the angles of a triangle is 180°.



Solution: Angle a is 35° because it alternates with the angle inside the triangle that measures 35°. Angle c because it alternates with the angle inside the triangle that measures 80°. Because lines have a measure of 1 angles $a + b + c$ form a straight line, then angle b must be 65 ° → 180 – (35 + 80) = 65. Therefore, the sum angles of the triangle is 35° + 65 ° + 80 °.

Example 4:

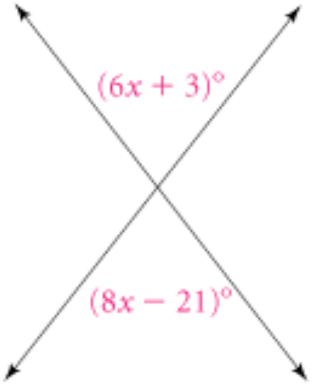
What is the measure of angle 5 if the measure of angle 2 is 45° and the measure of angle 3 is 60°?



Solution: Angles 2 and 4 are alternate interior angles, therefore the measure of angle 4 is also 45°. The measure of angles 3, 4 and 5 must add to 180°. If angles 3 and 4 add to 105° the angle 5 must be equal to 75°.

Students construct various triangles having line segments of different lengths but with two corresponding cc angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar. S solve problems with similar triangles.

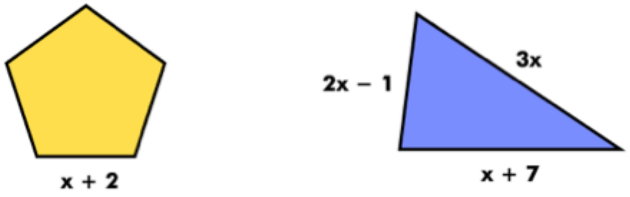
Geometry Find the value of x in the diagram below.



$6x + 3 = 8x - 21$	Vertical angles are congruent.
$6x + 3 - 6x = 8x - 21 - 6x$	Subtract 6x from each side.
$3 = 2x - 21$	Combine like terms.
$3 + 21 = 2x - 21 + 21$	Add 21 to each side.
$24 = 2x$	Simplify.
$\frac{24}{2} = \frac{2x}{2}$	Divide each side by 2.
$x = 12$	Simplify.

The value of x is 12.

The perimeter of these figures is equal — find the perimeter



$5(x + 2) = 2x - 1 + 3x + x + 7$

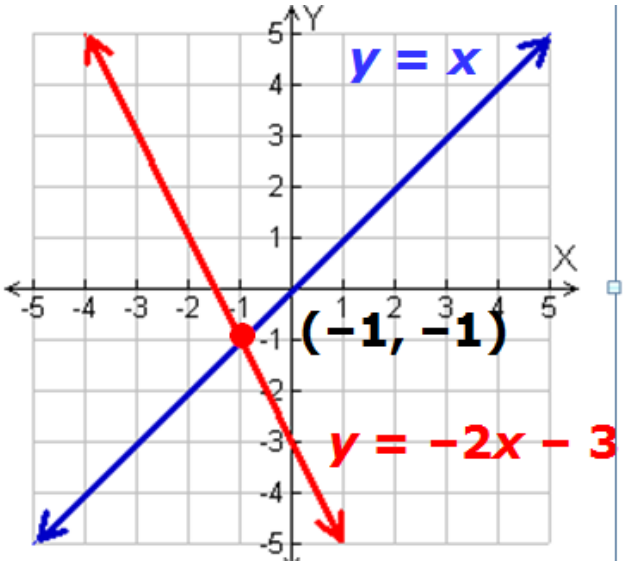
$5(x + 2) = 6x + 6$

$5x + 10 = 6x + 6$

$x = 4$

The perimeter is 30 units.

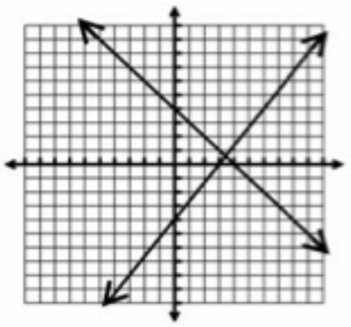
Expression, Equations and Relationships: **TEK 8.9**	**The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to:**
8.9(A) Identify and verify the values of *x* and *y* that simultaneously satisfy two linear equations in the form *y* = *mx* + *b* from the intersections of the graphed equations	Students need to know that intersection of two lines is a point that satisfies both equations. Students need to know that when you are referring to the intersection of two lines it is also called a system of equations. Students need to know that the intersection of the two lines is also called a solution. Students must be able to graph an equation in the form y=mx+b. **Common errors:** 1. Students sometimes do not substitute the (x,y) point correctly into the two equations. 2. Students do not graph coordinates/lines correctly. 3. Students use (y,x) instead of (x,y). **Examples:** **Check your answer. Find the solution of y=x and y = -2x-3.**

	Y X y = x (−1, −1) y = −2x − 3 -5 -4 -3 -2 -1 1 2 3 4 5 5 4 3 2 1 -1 -2 -3 -4 -5

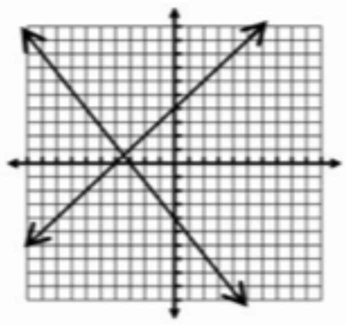
Which of the following graphs shows the solution to this system of linear equations?

$y = x + 4 \qquad y = -\frac{4}{3}x - 4$

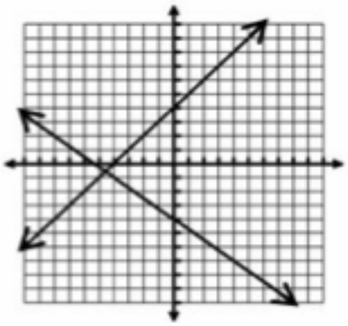
A



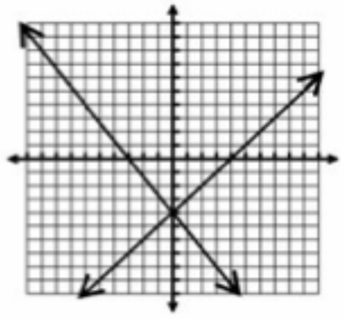
C

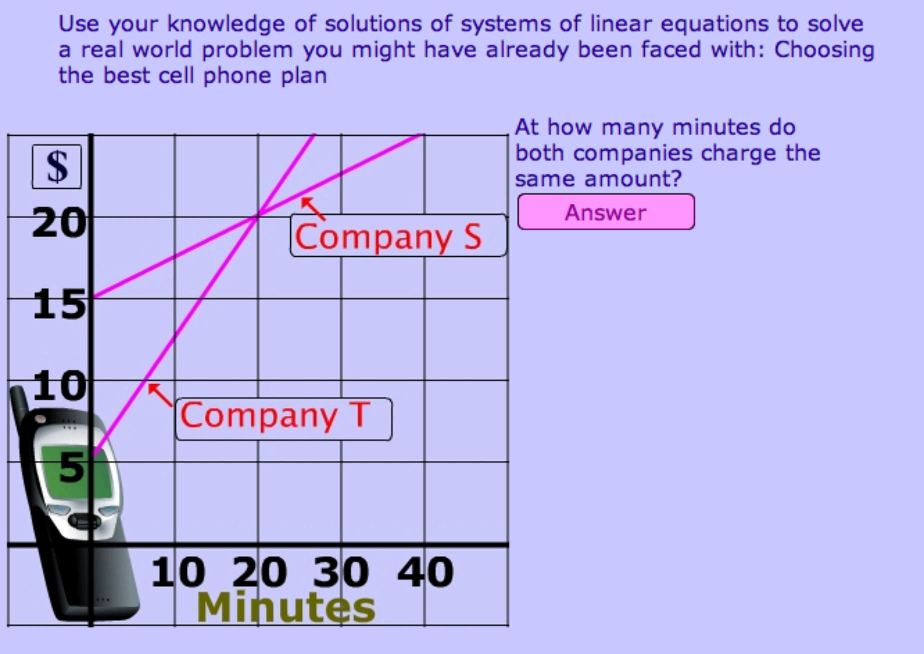


B



D



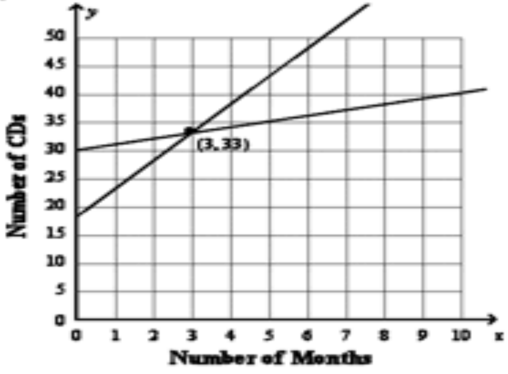


Use your knowledge of solutions of systems of linear equations to solve a real world problem you might have already been faced with: Choosing the best cell phone plan

At how many minutes do both companies charge the same amount?

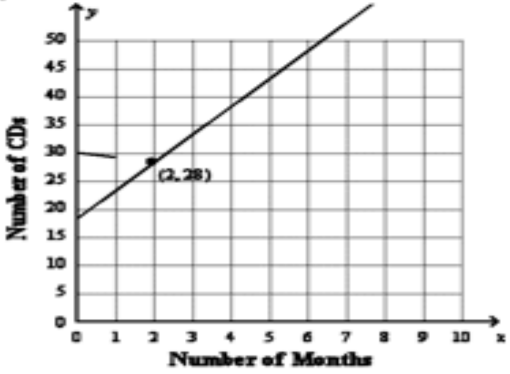
Tom has a collection of 30 CDs and Nita has a collection of 18 CDs. Tom is adding CD a month to his collection while Nita is adding 5 CDs a month to her collection. Write and graph a system to find the number of months after which they will have the same number of CDs. Let x represent the number of months and y the number of CDs.

A. $y = x + 30$
$y = 5x + 18$



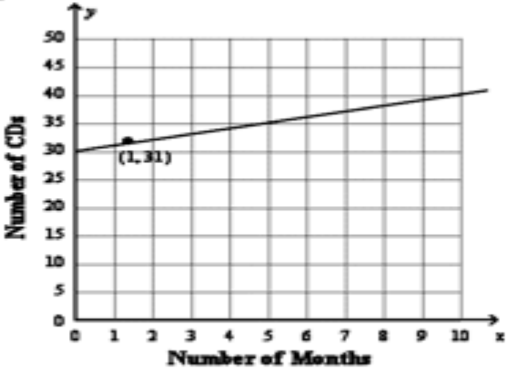
3 months

B. $y = -x + 30$
$y = 5x + 18$



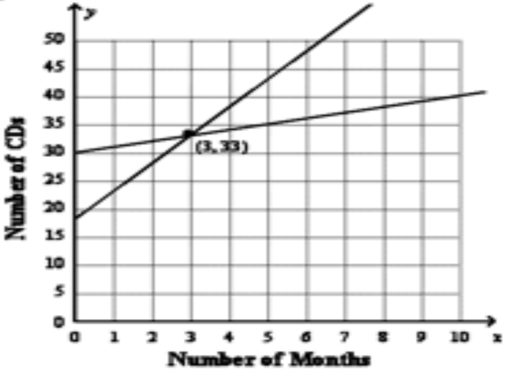
2 months

C. $y = x + 30$
$y = 18x + 5$



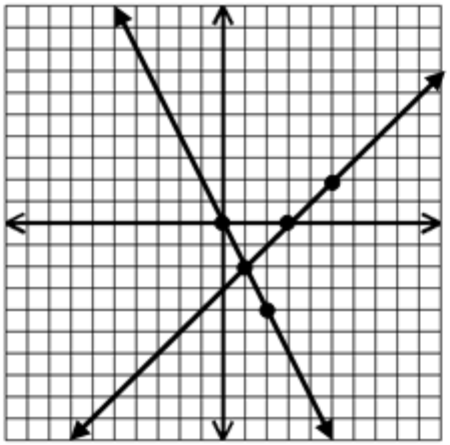
1 month

D. $y = x + 18$
$y = 5x + 30$



33 months

The solution to the system below is



A. (1, 2)

B. (-1, -2)

C. (-1, 2)

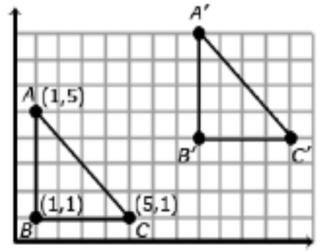
D. (1, -2)

Two-Dimensional Shapes: TEK 8.10	**The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:**
8.10(A) Generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane; 8.10(B) Differentiate between transformations that preserve congruence and those that do not. 8.10(C) Explain the effect of translations, reflections over the *x*- or *y*- *axis,* and rotations limited to 90°, 180°, 270°, and 360° as applied to two--dimensional shapes on a coordinate plane using an algebraic representation	Students recognize the relationship between the coordinates of the pre-image, the image and the scale factor for a dilation from the origin. Using the coordinates, students are able to identify the scale factor (image/pre-image). Students identify the transformation based on given coordinates. For example, the pre-image coordinates of a triangle are A(4, 5), B(3, 7), and C(5, 7). The image coordinates are A(-4, 5), B(-3, 7), and C(-5, 7). What transformation occurred? Students must understand that rotations, reflections and translations preserve congruence but dilations do not unless the scale factor is one. For *Clockwise* Rotations: algebraic rules 90 degrees: (x,y) goes to (y, -x) 180 degrees: (x,y) goes to (-x, -y) 270 degrees: (x,y) goes to (-y, x) 360 degress: (x,y) stays (x,y) 5. For Dilations: algebraic rule: (x,y) goes to (ax,ay) where a is the scale factor. 6. Algebraic rule for reflections across an axis: reflection across the y-axis: (x,y) goes to (-x,y) reflection across the x-axis: (x,y) goes to (x, -y) **Common Misconceptions:** Students confuse x and y axis.

Examples:

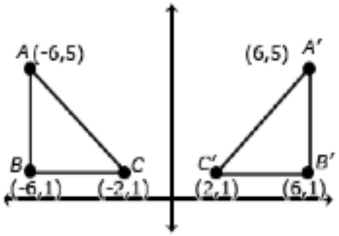
Translations

Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. Triangle ABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B and C also move in the same direction (7 units to the right and 3 units up), resulting in the same changes to each coordinate.



Reflections

A reflection is the "flipping" of an object over a line, known as the "line of reflection". In the 8th grade, the line of reflection will be the x-axis and the y-axis. Students recognize that when an object is reflected across the y-axis, the reflected x-coordinate is the opposite of the pre-image x-coordinate (see figure below).

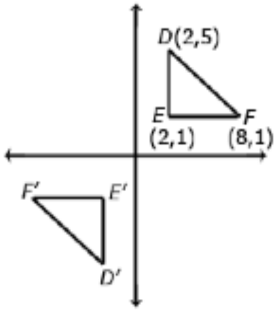


Likewise, a reflection across the x-axis would change a pre-image coordinate (3, -8) to the image coordinate of (3, 8) -- note that the reflected y-coordinate is opposite of the pre-image y-coordinate.

Rotations

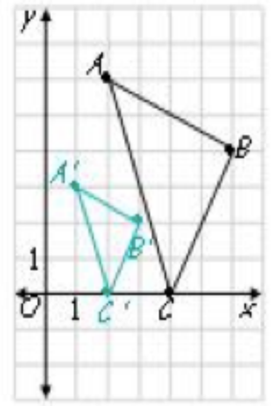
A rotation is a transformation performed by "spinning" the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to 360º (at 8th grade, rotations will be around the origin and a multiple of 90º). In a rotation, the rotated object is *congruent* to its pre-image

Consider when triangle DEF is 180˚ clockwise about the origin. The coordinate of triangle DEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚ about the origin, the new coordinates are D'(-2,-5), E'(-2,-1) and F'(-8,-1). In this case, each coordinate is the opposite of its pre-image (see figure below).



Dilations

A dilation is a non-rigid transformation that moves each point along a ray which starts from a fixed center, and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the scale factor. In 8th grade, dilations will be from the origin. The dilated figure is *similar* to its pre-image.

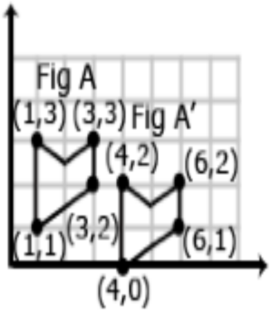


The coordinates of A are (2, 6); A' (1, 3). The coordinates of B are (6, 4) and B' are (3, 2). The coordinates of C are (4, 0) and C' are (2, 0). Each of the image coordinates is ½ the value of the pre-image coordinates indicating a scale factor of ½.

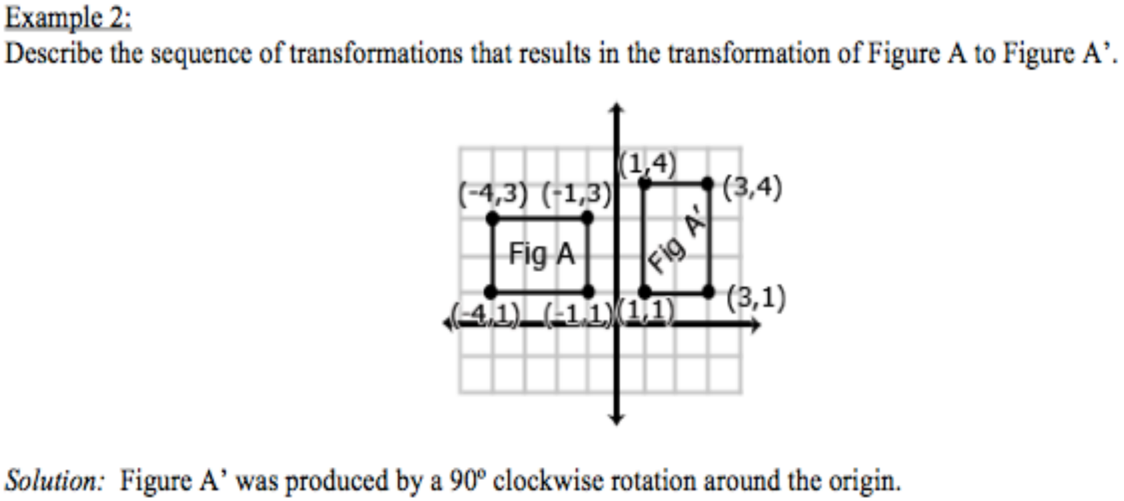
The scale factor would also be evident in the length of the line segments using the ratio: $\frac{\text{image length}}{\text{pre-image length}}$

Example 1:

Is Figure A congruent to Figure A'? Explain how you know.



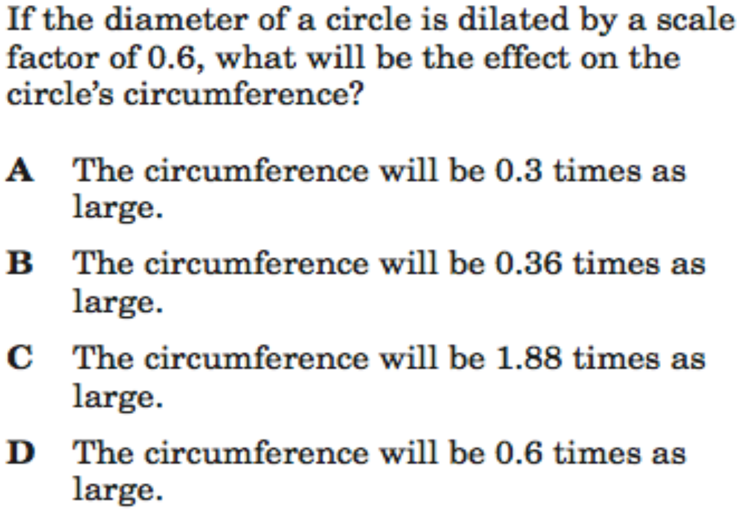
Solution: These figures are congruent since A' was produced by translating each vertex of Figure A 3 to the right and 1 down

	 Example 2: Describe the sequence of transformations that results in the transformation of Figure A to Figure A'. *Solution:* Figure A' was produced by a 90º clockwise rotation around the origin.
8.10(D) Model the effect on linear and area measurement of dilated two-dimensional shapes.	Student needs to understand that perimeter, including circumference, is a linear measurement. Student needs to understand that when an image is dilated, its' perimeter is just multiplied by the scale factor. Student understands that area is a two dimensional measurements (length and width) using square units. Student needs to understand that when an image is dilated the area of the new image is the the product of the area of the original figure and the scale factor squared. Students understand that a dilation with scale factor greater than 1 is an enlargement and a scale factor less than one is a reduction. Students should also understand that the scale factor from one image to another is the reciprocal if reversing the dilation. **Common Errors:**

1. Applying scale factor once on area problems, instead of twice for each dimension.
2. Students will assume that an improper fraction as the scale factor is a reduction instead of an enlargement.

Examples:

1. Have student draw a rectangle on the coordinate plane. Have them dilate the rectangle using a scale factor of 3. Have them calculate the perimeter and area of both original and dilated images and describe the change. (can extend this example to drawing a circle with a given radius)



If the diameter of a circle is dilated by a scale factor of 0.6, what will be the effect on the circle's circumference?

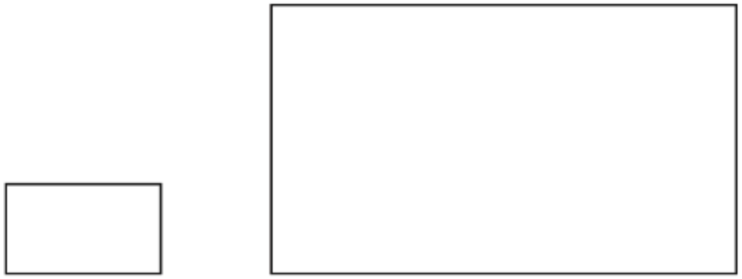
A The circumference will be 0.3 times as large.

B The circumference will be 0.36 times as large.

C The circumference will be 1.88 times as large.

D The circumference will be 0.6 times as large.

The two rectangles shown below are similar. The ratio of the length of the larger rectangle to the length of the smaller rectangle is 3:1.



Which of the following statements is true?

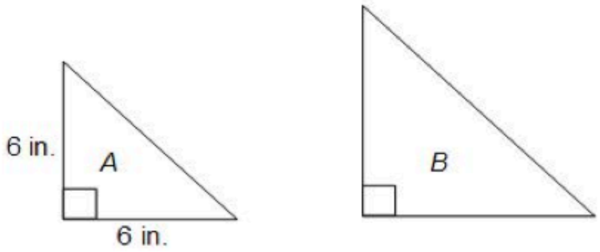
F The area of the smaller rectangle is $\frac{1}{4}$ the area of the larger rectangle.

G The area of the larger rectangle is 3 times the area of the smaller rectangle.

H The perimeter of the smaller rectangle is $\frac{1}{3}$ the perimeter of the larger rectangle.

J The perimeter of the larger rectangle is 6 times the perimeter of the smaller rectangle.

Triangle *A* is similar to triangle *B*. A scale factor of $\frac{4}{3}$ is applied to the length of the legs of triangle *A* to create triangle *B*.



What is the area of triangle *B*?

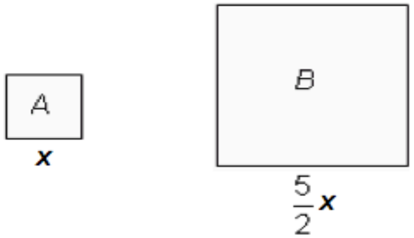
A 18 square inches

B 32 square inches

C 36 square inches

D 64 square inches

Squares *A* and *B* are similar.



If the area of square *A* is 25 square centimeters, what is the area of square *B*?

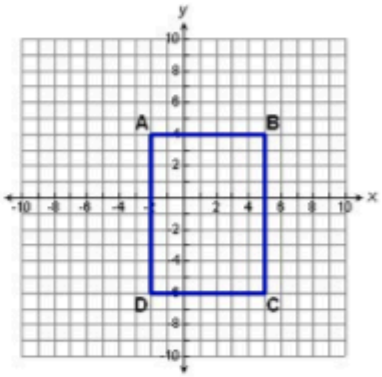
A 12.5 cm^2

B 56.25 cm^2

C 156.25 cm^2

D 3906.25 cm^2

Rectangle *ABCD* is shown on the coordinate grid below.



If a scale factor of $\frac{1}{2}$ each side to create rectangle *PQRS*, what will be the perimeter of rectangle *PQRS*?

A 5 units

B 17 units

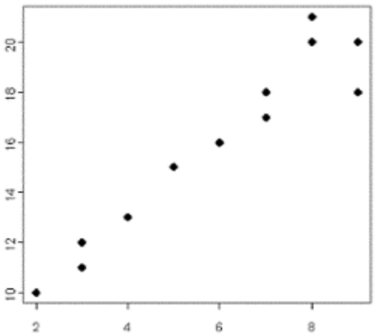
C 34 units

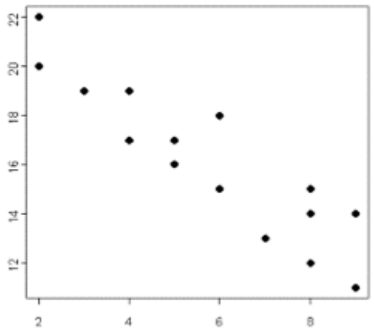
D 68 units

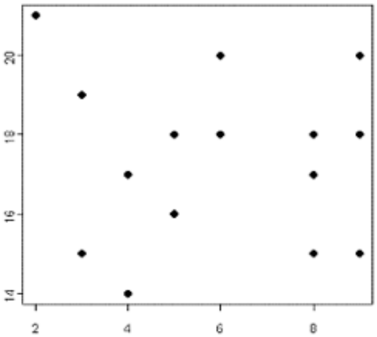
The length of each side of a right triangle is multiplied by 3. What effect does this have on the area of a triangle?

Describe the effect on the area of a circle when the radius is doubled.

Measurement and Data: TEK 8.11	**The student applies mathematical process standards to use statistical procedures to describe data. The student is expected to:**
8.11(A) Construct a scatterplot and describe the observed data to address questions of association such as linear, non---linear, and no association between bivariate data;	Students represent numerical data on a scatter plot, to examine relationships between variables. Students analyze scatter plots to determine if the relationship is linear (positive, negative trend or no trend) or non- linear. Students must understand that in a positive relationship, as one set of data increases, so does the other. The opposite, as one set of data decreases so does the other also represents a positive relationship. Students must understand that in a negative relationship, as one set of data increases the other must decrease and vice versa. "Trend" and "correlation" are interchangeable in reference to scatterplots. **Common Misconceptions:** 1. Just because the points are not perfectly lined up does not mean that the data cannot be represented as a linear relationship. 2. Students think that if both numbers in the data are decreasing, then it represents a negative trend. 3. Students think that there is no correlation if the x value is not in numeric order. A scatterplot is often employed to identify potential trends between two variables. A **positive trend** between two variables would be indicated on a scatterplot by an upward trend (positive slope) where both variables are increasing or both decreasing. A **negative trend** would be indicated by the opposite effect (negative slope), where the one variable increases while the other decreases, or vice versa. Or, there might not be any notable association, in which case a scatterplot would not indicate any trends whatsoever. The following plots demonstrate the appearance of positively associated, negatively associated, and non-associated variables:







Data can be expressed in years. In these situations it is helpful for the years to be "converted" to 0, 1, 2, etc. F example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 198

Example 1:
Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

Solution: This data has a positive association.

Example 2:
Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Distance from School (miles)	0.5	1.8	1	2.3	3.4	0.2	2.5	1.6	0.8	2.5

Solution: There is no association between the math score and the distance a student lives from school.

Example 3:
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of Staff	3	4	5	6	7	8
Average time to fill order (seconds)	56	24	72	63	42	93

Solution: There is a positive association.

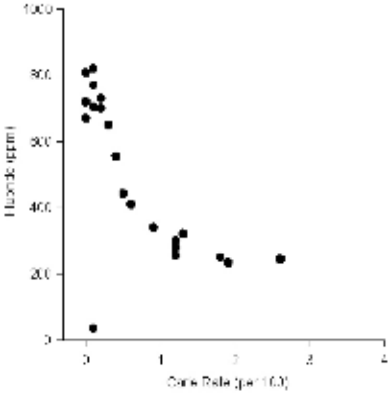
Example 4:
The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Solution: There is a positive association.

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model.

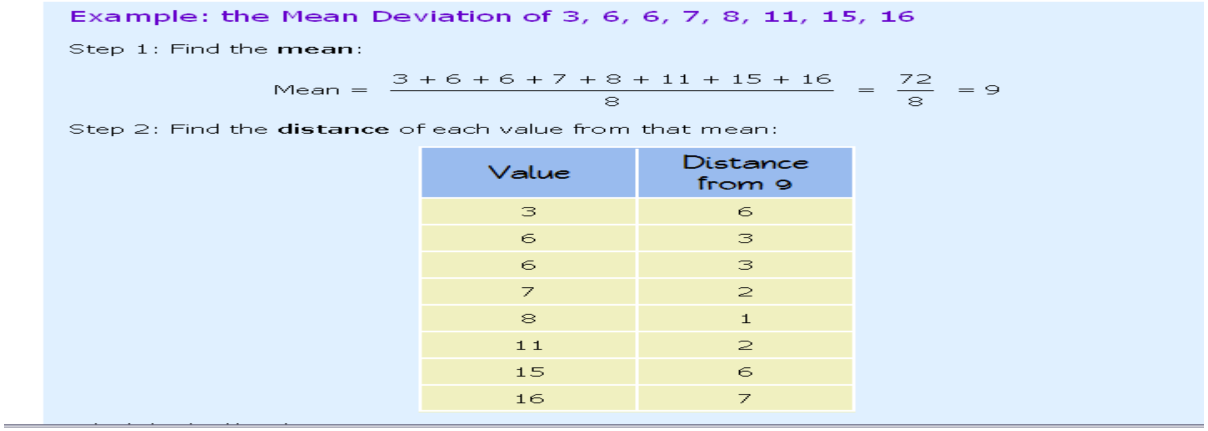
Students recognize that not all data will have a linear association. Some associations will be non-linear as in the example below:



8.11(B) Determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points; and

Student understands that the mean absolute deviation gives the average variation of the data from the mean. Student understands that the mean absolute deviation describes the dispersion or spread of data. It tells us how far, on average, all values are from the middle.
The student must be able to find the mean from a given set of data.

1. Calculate the mean of your data set.
2. Subtract the mean from each of the data values and list the differences (take absolute value of these differences so that you do not have negative numbers).
3. Then find the mean of those differences (distance).



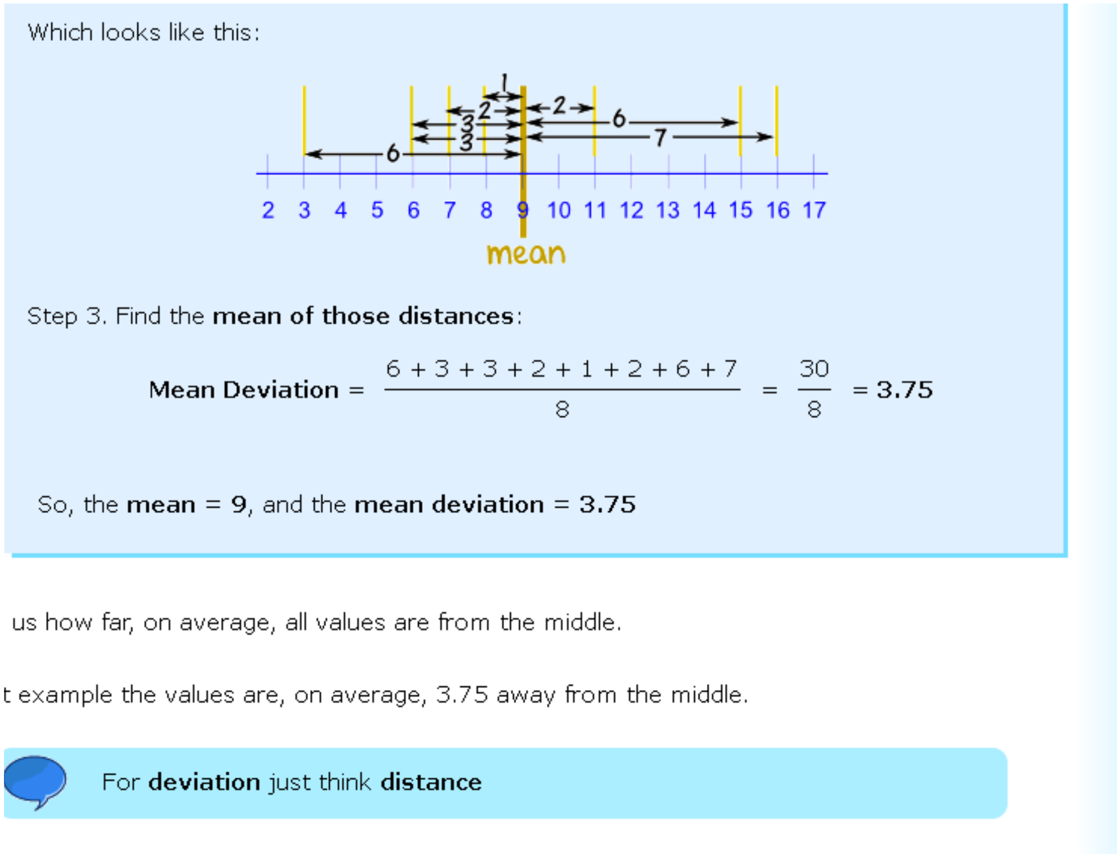
Example: the Mean Deviation of 3, 6, 6, 7, 8, 11, 15, 16

Step 1: Find the **mean**:

$$\text{Mean} = \frac{3 + 6 + 6 + 7 + 8 + 11 + 15 + 16}{8} = \frac{72}{8} = 9$$

Step 2: Find the **distance** of each value from that mean:

Value	Distance from 9
3	6
6	3
6	3
7	2
8	1
11	2
15	6
16	7



Which looks like this:

Step 3. Find the **mean of those distances**:

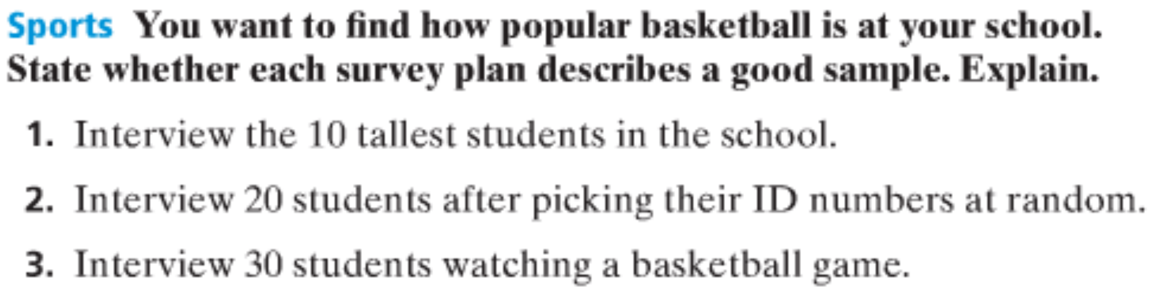
$$\textbf{Mean Deviation} = \frac{6 + 3 + 3 + 2 + 1 + 2 + 6 + 7}{8} = \frac{30}{8} = \mathbf{3.75}$$

So, the **mean** = **9**, and the **mean deviation** = **3.75**

us how far, on average, all values are from the middle.

t example the values are, on average, 3.75 away from the middle.

For **deviation** just think **distance**

8.11(C) Simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.	Student needs to know the meaning of random, sample, population, bias. Student needs to explain why a sample is a good/bad representation of the population. **Common misconceptions.** 1. Sometimes students will confuse the group that is being surveyed (sample space) with the population. **Examples:**  **Sports** **You want to find how popular basketball is at your school. State whether each survey plan describes a good sample. Explain.** **1.** Interview the 10 tallest students in the school. **2.** Interview 20 students after picking their ID numbers at random. **3.** Interview 30 students watching a basketball game.

Personal Financial Literacy: **TEK 8.12**	**The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:**
8.12(A) Solve real-world problems comparing how interest rates and loan length affect the cost of credit;	Students will need to be able to find interest on loans using several different interest rates. Students will need to compare the rates and how that affects the total cost of the loan. Students will need to compare the length of time of a loan and determine how that affects the total cost of the loan. Resource: http://www.handsonbanking.org/en/ Anthony has a balance of $10,000 on his credit card and the interest rate is 19%. Answer the following questions about Anthony's credit card situation. 1. To pay off the $10,000.00 in three years, Anthony will have to make a minimum payment of $365.56 per month. How much will Anthony pay in interest? 2. To pay off the $10,000.00 in five years, Anthony will have to make a minimum payment of $259.41 per month. How much will Anthony pay in interest? 3. How much more did Anthony have to pay when the length of the loan changed from 3 years to 5 years? Brianna also has a balance of $10,000 on her credit card. After talking to a financial advisor, Brianna decided to get a loan from the bank and discontinue using her credit card. The bank will charge Brianna 4% interest. 4. To pay off the $10,000 loan in three years, Brianna will have to pay $295.24 per month. How much will Brianna pay in interest?

5. To pay off the $10,000 loan in five years, Brianna will have to pay $184.17 per month. How much will Brianna pay in interest?

6. How much more did Brianna have to pay when the length of the loan changed from 3 years to 5 years?
7. Now compare Anthony's interest paid to Brianna's interest paid for the three year loan. Who paid the most interest? How much more?

8. When comparing Anthony's five year loan to Brianna's 5 year loan, who paid the most interest? How much more?

9. After studying Anthony's case and Brianna's case, what factor had the greatest impact on reducing the total payment?

Extension: Have the students do the same exercise using an on-line credit card calculator. The monthly loan payments have been left blank as to allow students to calculate the monthly payment.
http://www.bankrate.com/

Anthony has a balance of $10,000 on his credit card and the interest rate is 19%. Answer the following questions about Anthony's credit card situation.

1. To pay off the $10,000.00 in three years, Anthony will have to make a minimum payment of $__________ per month. How much will Anthony pay in interest?

2. To pay off the $10,000.00 in five years, Anthony will have to make a minimum payment of $__________per month. How much will Anthony pay in interest?

3. How much more did Anthony have to pay when the length of the loan changed from 3 years to 5 years?

	Brianna also has a balance of $10,000 on her credit card. After talking to a financial advisor, Brianna decided to get a loan from the bank and discontinue using her credit card. The bank will charge Brianna 4% interest. 4. To pay off the $10,000 loan in three years, Brianna will have to pay $__________ per month. How much will Brianna pay in interest? 5. To pay off the $10,000 loan in five years, Brianna will have to pay $__________ per month. How much will Brianna pay in interest? 6. How much more did Brianna have to pay when the length of the loan changed from 3 years to 5 years? 7. Now compare Anthony's interest paid to Brianna's interest paid for the three year loan. Who paid the most interest? How much more? 8. When comparing Anthony's five year loan to Brianna's five year loan, who paid the most interest? How much more? 9. After studying Anthony's case and Brianna's case, what factor had the greatest impact on reducing the total payment? Sample Lessons: http://smartertexas.org/?page_id=914
8.12(B) Calculate the total cost of repaying a loan, including credit cards and over different periods using an online	Students use online resources to calculate loans with compound interest. Students will need to calculate the total cost of purchasing items on a credit card (and other loans like car loans) using different interest rates and different lengths of time (12 months, 24 months, 36 months, etc). Students will understand that the principle decreases over time.

<table>
<tr>
<td>calculator.</td>
<td>
Additional Resources:
http://www.stlouisfed.org/education_resources/assets/lesson_plans/paycheck/lesson_8_your_paycheck%20lp.pdf
http://www.stlouisfed.org/education_resources/assets/lesson_plans/paycheck/IYP_lesson8.pdf
http://www.stoppaydayabuse.org/

1. Use the on-line calculator below to calculate the total repayment of each loan and the interest paid.

On-line calculator: http://www.bankrate.com/calculators/mortgages/loan-calculator.aspx

<table>
<tr>
<td>A</td>
<td>Loan Amount: $6000
Loan Term: 3 years
Interest Rate: 6%</td>
<td>Monthly Payment: ______________
Total Repayment: _______________
Interest Paid: ___________</td>
</tr>
<tr>
<td>B</td>
<td>Loan Amount: $6000
Loan Term: 4 years
Interest Rate: 6%</td>
<td>Monthly Payment: ______________
Total Repayment: _______________</td>
</tr>
</table>
</td>
</tr>
</table>

		Interest Paid: ____________
C	Loan Amount: $6000 Loan Term: 3 years Interest Rate: 5%	Monthly Payment: _______________ Total Repayment: ________________ Interest Paid: ____________
D	Loan Amount: $6000 Loan Term: 4 years Interest Rate: 5%	Monthly Payment: _______________ Total Repayment: ________________ Interest Paid: ____________

2. Compare the difference of interest paid when the number of years to repay the loan remained constant.
   a. A and C __________
   b. B and D __________

3. Compare the difference of interest paid when the interest rate remains constant and the number of years increases.
   a. A and B _________
   b. C and D ________

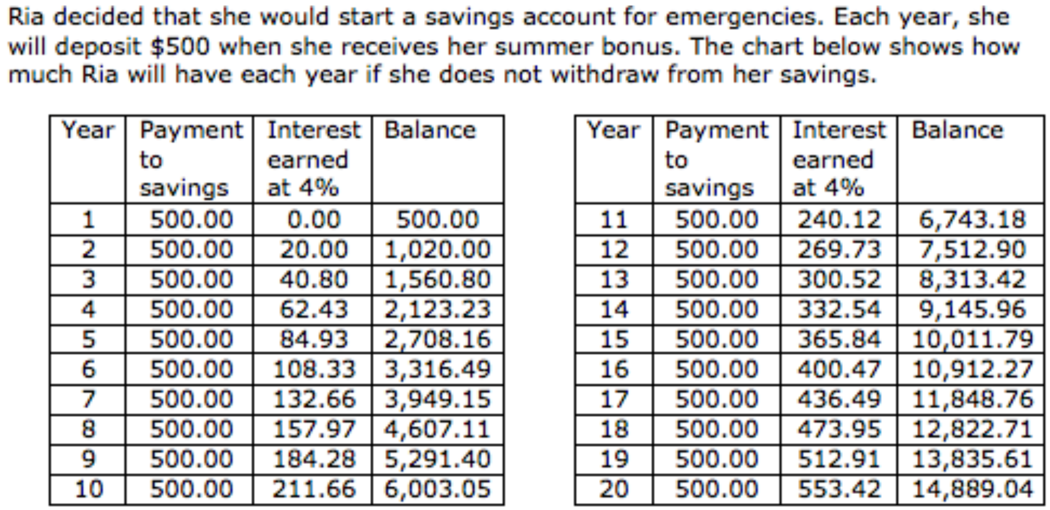
4. If you could only budget a maximum of $150 per month, which option would be your choice? Explain your choice.

Sample Lessons:
http://smartertexas.org/?page_id=914

8.12(C) Explain how small amounts of money invested regularly, including money saved for college and retirement, grow over time.

Students will need to understand that the principle balance grows (changes) as interest is earned. Students will compare the different amounts of interest earned over time and what that does to the balance.

Additional Resources http://www.scholastic.com/smp/pdfs/mint/mint_twp_web_worksheet2.pdf



Ria decided that she would start a savings account for emergencies. Each year, she will deposit $500 when she receives her summer bonus. The chart below shows how much Ria will have each year if she does not withdraw from her savings.

Year	Payment to savings	Interest earned at 4%	Balance
1	500.00	0.00	500.00
2	500.00	20.00	1,020.00
3	500.00	40.80	1,560.80
4	500.00	62.43	2,123.23
5	500.00	84.93	2,708.16
6	500.00	108.33	3,316.49
7	500.00	132.66	3,949.15
8	500.00	157.97	4,607.11
9	500.00	184.28	5,291.40
10	500.00	211.66	6,003.05
11	500.00	240.12	6,743.18
12	500.00	269.73	7,512.90
13	500.00	300.52	8,313.42
14	500.00	332.54	9,145.96
15	500.00	365.84	10,011.79
16	500.00	400.47	10,912.27
17	500.00	436.49	11,848.76
18	500.00	473.95	12,822.71
19	500.00	512.91	13,835.61
20	500.00	553.42	14,889.04

1. How much interest did Ria earn in the first five years?
2. How much interest did Ria earn from year 6 to year 10?
3. How much interest did Ria earn from year 11 to year 15?
4. How much interest did Ria earn from year 16 to year 20?

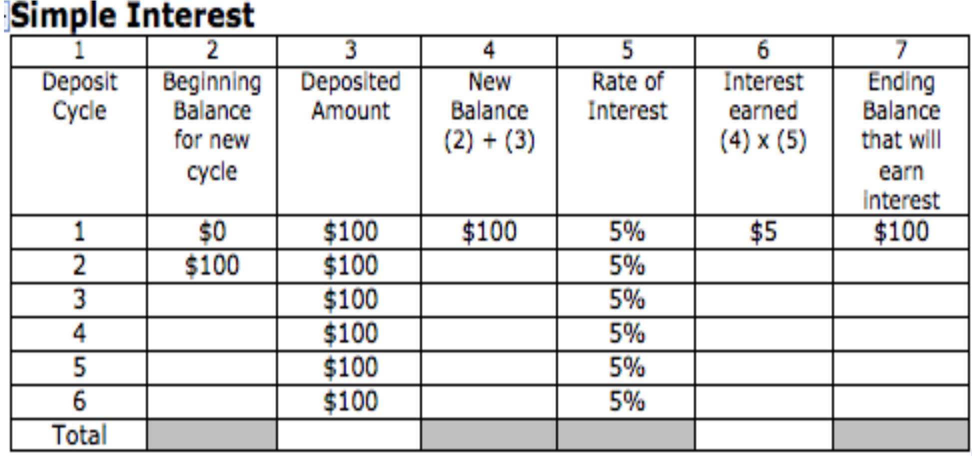
5. Describe how the interest from each 5-year interval compared?
6. What factors caused each set of numbers to change?
7. If Ria continues to invest $500 each year for 5 years and does not withdraw any money from her savings account, which of the following could be her balance? $16,000, $18,000, $20,000, or $25,000

Sample Lessons:
http://smartertexas.org/?page_id=914

8.12(D) Calculate and compare simple interest and compound earnings.

Students will calculate simple and compound interest using various rates and time periods. Students will compare the different interest earnings from simple and compound.

Emma opened a savings account that will pay 5% simple interest each year. She will deposit $100 each year on her birthday. Use the chart below to calculate the interest she will earn over 6 years.

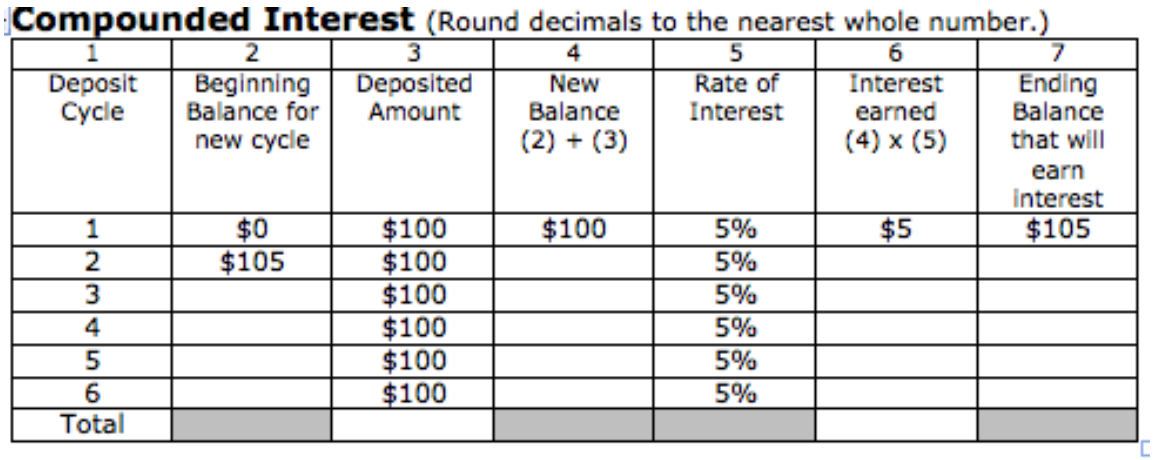


Simple Interest

1 Deposit Cycle	2 Beginning Balance for new cycle	3 Deposited Amount	4 New Balance (2) + (3)	5 Rate of Interest	6 Interest earned (4) x (5)	7 Ending Balance that will earn interest
1	$0	$100	$100	5%	$5	$100
2	$100	$100		5%		
3		$100		5%		
4		$100		5%		
5		$100		5%		
6		$100		5%		
Total						

2. How much will Emma have in her account after 6 years?

3. Ethan also opened a savings account that will pay 5% compounded annually. He will deposit $100 each year on his birthday. Use the chart below to calculate the interest he will earn over 6 years.



Compounded Interest (Round decimals to the nearest whole number.)

1	2	3	4	5	6	7
Deposit Cycle	Beginning Balance for new cycle	Deposited Amount	New Balance (2) + (3)	Rate of Interest	Interest earned (4) x (5)	Ending Balance that will earn interest
1	$0	$100	$100	5%	$5	$105
2	$105	$100		5%		
3		$100		5%		
4		$100		5%		
5		$100		5%		
6		$100		5%		
Total						

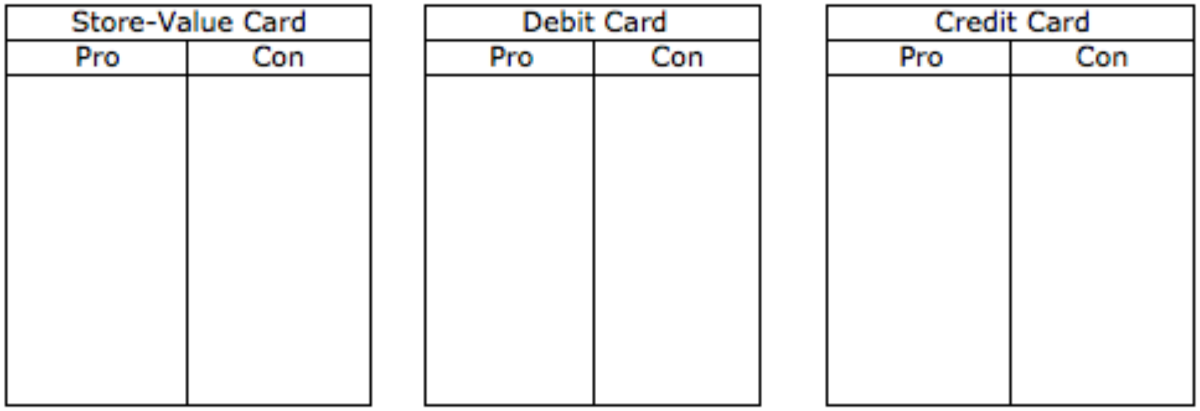
4. How much will Ethan have in his account in 6 years?

5. Explain the differences between Emma's savings plan and Ethan's savings plan.

Adapted from: http://ecedweb.unomaha.edu/lessons/M&M6-8.pdf
Additional Resource: http://www.winthemoneygame.com/money-game/simple-vs-compound-interest-activity-lesson/
https://www.mymetrobank.com/filestore/Grade6-8BudgetingandSavingforaSpecialPurpose.pdf
http://www.frbatlanta.org/documents/forms/katrina/lessonplans/3.pdf

Sample Lessons:

	http://smartertexas.org/?page_id=914
8.12(E) Identify and explain the advantages and disadvantages of different payment methods	Students will understand the different options for purchasing items. Students will need to compare the options and determine the best one to use in different real-life experiences. And students will need to understand the consequences of using an interest charging credit card versus a debit card. The three descriptions below describe 3 options for purchasing an item. Stored-value card (Sometimes called a gift card) - Anyone who makes purchases with a merchant gift card, places phone calls with a prepaid telephone card, or buys goods or services with a prepaid card is using a stored- value card. *Source: Federal Reserve of New York* Debit Card - An electronic card issued by a bank which allows bank clients access to their account to withdraw cash or pay for goods and services. This removes the need for bank clients to go to the bank to remove cash from their account as they can now just go to an ATM or pay electronically at merchant locations. This type of card, as a form of payment, also removes the need for checks as the debit card immediately transfers money from the client's account to the business account. *Source: Investopedia* Credit Card - A credit card is a plastic payment card often issued by a bank. It authorizes the delivery of goods and services in exchange for future payment with interest. Customers receive a monthly bill and may be charged a yearly fee. *Source: Federal Reserve of Richmond* 1. Have students research the three cards on the internet. Or have a guest speaker from a local financial institution talk about the three cards. 2. Then have the students list the similarities and differences between the three cards. Scaffolding questions: What are the physical similarities or differences? What type of information is on each card? When a card is used, where is the money deducted from? Which card can access cash? Which cards have fees? 3. Finally have the students list pros and cons for using each type of card.

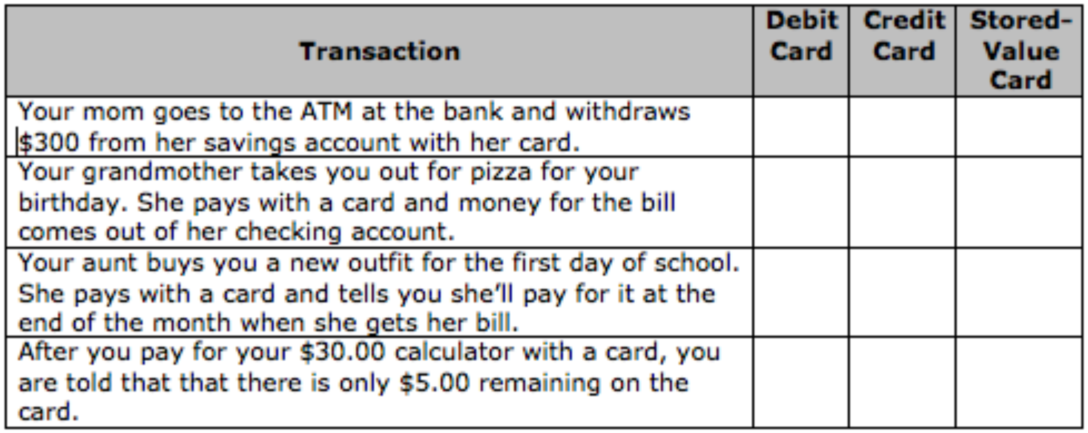


Store-Value Card	
Pro	Con

Debit Card	
Pro	Con

Credit Card	
Pro	Con

4. Look at the transaction in the first column below. Decide whether a debit card, a credit card, or a stored-value card was used for the transaction and put an X in the correct column.



Transaction	**Debit Card**	**Credit Card**	**Stored-Value Card**
Your mom goes to the ATM at the bank and withdraws $300 from her savings account with her card.			
Your grandmother takes you out for pizza for your birthday. She pays with a card and money for the bill comes out of her checking account.			
Your aunt buys you a new outfit for the first day of school. She pays with a card and tells you she'll pay for it at the end of the month when she gets her bill.			
After you pay for your $30.00 calculator with a card, you are told that that there is only $5.00 remaining on the card.			

	Adapted from: http://www.richmondfed.org/education/for_teachers/lesson_plans_and_classroom_activities/pdf/debit_credit_lesson.pdf Additional Resources: http://www.ny.frb.org/regional/stored_value_cards.html http://www.investopedia.com/terms/d/debitcard.asp#axzz1rBovsW00 http://www.richmondfed.org/education/for_teachers/lesson_plans_and_classroom_activities/pdf/debit_credit_lesson.pdf http://teacher.scholastic.com/lessonplans/moneyinmotion/lesson2.htm http://teacher.scholastic.com/lessonplans/moneyinmotion/pdfs/credit.pdf Sample Lessons: http://smartertexas.org/?page_id=914
8.12(F) Analyze situations to determine if they represent a financially responsible decision and identify the benefits of financial responsibility and the costs of financial irresponsibility	Students review different financial scenarios and analyze the decisions that were made. Students will need to compare and contrast the decisions and decide on what the best decision should have been. For each situation, examine the decision made by the person. Then determine if the person made a responsible decision. Explain your reasoning. a. Penny is buying a used car for $6000. She has $2000 in her savings. She decides to borrow the entire $6000 at 5% and use her savings to go on a vacation. b. Samuel's mother gave him $5.00 to buy the 20 ounce box of WakeUp Cereal for 3.99. When he got to the store, Samuel decided to buy two boxes of the 11 ounce cereal for $1.90 each. 2. Dillon is saving to buy a laptop that cost $900. So far he has saved $680.00 in his savings account. He has been able to deposit $75.00 every month into this savings. While reading the newspaper ads, he noticed that the computer is now on sale for 20% off. What financial advice would you give Dillon? http://www.scholastic.com/smp/pdfs/mint/mint_twp_web_worksheet1.pdf

	Sample Lessons: http://smartertexas.org/?page_id=914
8.12(G) Estimate the cost of a 2-year and 4-year college education including family contribution and devise a periodic savings plan for accumulating the money needed to contribute to the total cost of attendance for at least the 1st year of college.	Students will research/estimate the cost of attending a 2 year and 4 year college. Students will need to create a savings plan to determine how much money would need to be saved each year and how many years would be needed to collect enough money to pay for one full year of college. Note to teacher: For this activity, students will need internet access and they will need to download the Savings Estimator. Choose a College 1. Have you thought about which college you would like to attend? Explore the College For All Texans website http://www.collegeforalltexans.com/ to learn about colleges in Texas. Start with the College Match Up on the right column. Then explore the other features of the website. Spend time showing your parents the informational components of College For All Texans. 2. Use the State's Net Price Calculator to explore the situation below. a. Johnny would like to go to one the following colleges: Texas A&M Kingsville, University of Texas at Brownsville, or Texas A&M Corpus Christi. b. He would like to live on campus. c. He is 18 years old. d. He is not married. e. He has no dependents. f. He is a US citizen. g. He lives in a single parent household. h. His mother's annual salary is $41,340.00 i. His mother paid $1495.00 in income tax. j. Johnny made $1700 working part time in the summer. k. He paid no income tax. l. There are 4 people in Johnny's household. m. Johnny will be the only person in college.

3. What is the total cost for each university?
Texas A&M Kingsville _______________
University of Texas at Brownsville _______________
Texas A&M Corpus Christi ______________

4. What was the tuition for each university?
Texas A&M Kingsville _______________
University of Texas at Brownsville _______________
Texas A&M Corpus Christi ______________

Look at #5, this is the estimated grant and/or scholarship assistance. This is the assistance Johnny can expect to receive. Number 6 is what Johnny can expect to pay after grants and scholarships. Number 7 is the estimated student loans and/or student work earnings. Finally, number 8 represents the amount Johnny should have for each year he is in school.

Note: *For any line that gives a range of values, use the midpoint of that range when answering the following questions.*

5. Choose one of the three universities for Johnny. ________________________

6. What is the total cost for the university you choose for Johnny?

7. What is the estimated grant and/or scholarship assistance Johnny might receive from that college?
____

8. What is the estimated net cost? _______________

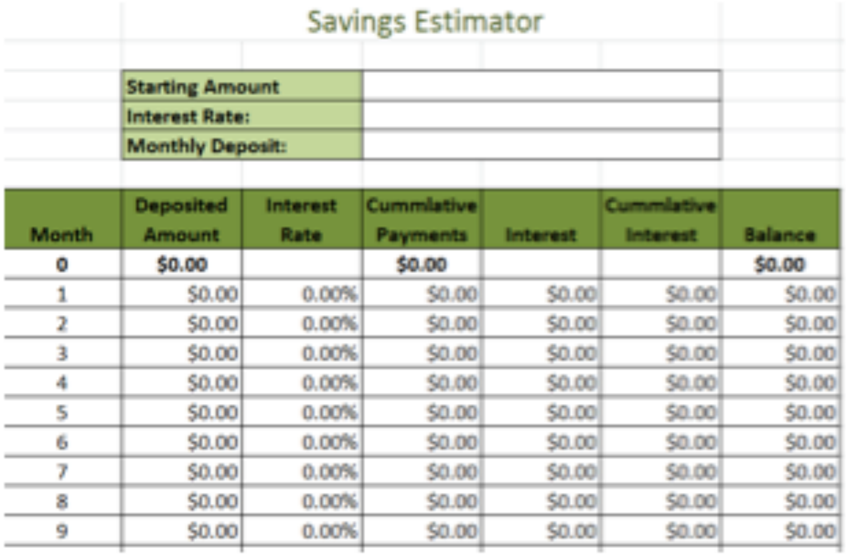
9. What is the estimate of the loans and work earnings Johnny may receive?

10. After the grant and/or scholarship assistance and loan and/or work earnings are subtracted from the total cost, how much will Johnny need for his first year of college (line 8)? ________________

Goal: Ideally, Johnny should have saved enough money to cover the total cost for at least 1 year at this university (line 8). Next you will determine how much Johnny should have saved monthly using the College Savings Estimator. This college savings estimator assumes that the interest rate remains constant and is compounded annually. It can also calculate up to 84 months.

11. Let's assume at age 11 Johnny's parents began making monthly payments to his college savings. And they continued making these payments until he reached the age of 18. How many months did his parents make payments to his college savings? _______________

12. Johnny had $1200 in his college savings before age 11 and the interest rate was 3.5% for each year. Enter these values into table 1 of the College Savings Estimator. Then predict how much Johnny should have saved monthly to reach his goal. Enter your prediction into row 3 of table 1. Now look at the month in which Johnny will turn 18. Is this number equal to what Johnny needs to meet his goal? If not, continue making prediction for the Monthly Payment till you reach a value close Johnny's goal.



Savings Estimator

Starting Amount	
Interest Rate:	
Monthly Deposit:	

Month	Deposited Amount	Interest Rate	Cummlative Payments	Interest	Cummlative Interest	Balance
0	$0.00		$0.00			$0.00
1	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
2	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
3	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
4	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
5	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
6	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
7	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
8	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00
9	$0.00	0.00%	$0.00	$0.00	$0.00	$0.00

Table 1

13. How would the cost change, if Johnny stayed at home the first two years and went to Victoria College before going to a university? Use the Net's Price Calculator to determine the cost for attending Victoria College while Johnny lives at home.

	Sample Lessons: http://smartertexas.org/?page_id=914 Extension: · Have students explore other options for saving using the Compounding Calculator at http://www.themint.org/kids/compounding-calculator.html. · For homework, have students demonstrate how to use the State Net Price Calculator with their parents. Resource: http://www.collegeforalltexans.com/ http://www.finaid.org/loans/