UNIT 1

Numbers

Contents

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Unit Pacing Guide

45-Minute Classes

Module 1

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Lesson 1.1	Lesson 1.1	Lesson 1.2	Lesson 1.2	Lesson 1.3
DAY 6	**DAY 7**			
Lesson 1.3	Ready to Go On? Texas Test Prep			

Module 2

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Lesson 2.1	Lesson 2.1	Lesson 2.2	Lesson 2.2	Lesson 2.3
DAY 6	**DAY 7**			
Lesson 2.3	Ready to Go On? Texas Test Prep			

90-Minute Classes

Module 1

DAY 1	DAY 2	DAY 3	DAY 4
Lesson 1.1	Lesson 1.2	Lesson 1.3	Ready to Go On? Texas Test Prep

Module 2

DAY 1	DAY 2	DAY 3	DAY 4
Lesson 2.1	Lesson 2.2	Lesson 2.3	Ready to Go On? Texas Test Prep

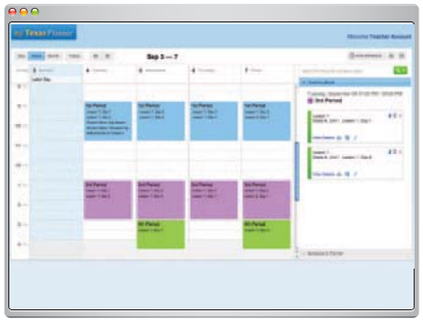
Program Resources

Plan

Online Teacher Edition

Access a full suite of teaching resources online—plan, present, and manage classes, assignments, and activities.

ePlanner **Easily plan your classes, create and view assignments, and access all program resources with your online, customizable planning tool.**



Professional Development Videos

Author Juli Dixon models successful teaching practices and strategies in actual classroom settings.

QR Codes **Scan with your smart phone to jump directly from your print book to online videos and other resources.**

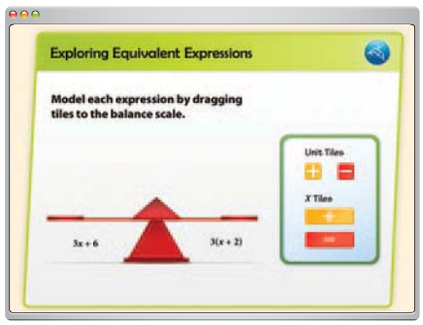
Teacher's Edition

Support students with point-of-use Questioning Strategies, teaching tips, resources for differentiated instruction, additional activities, and more.

Engage and Explore

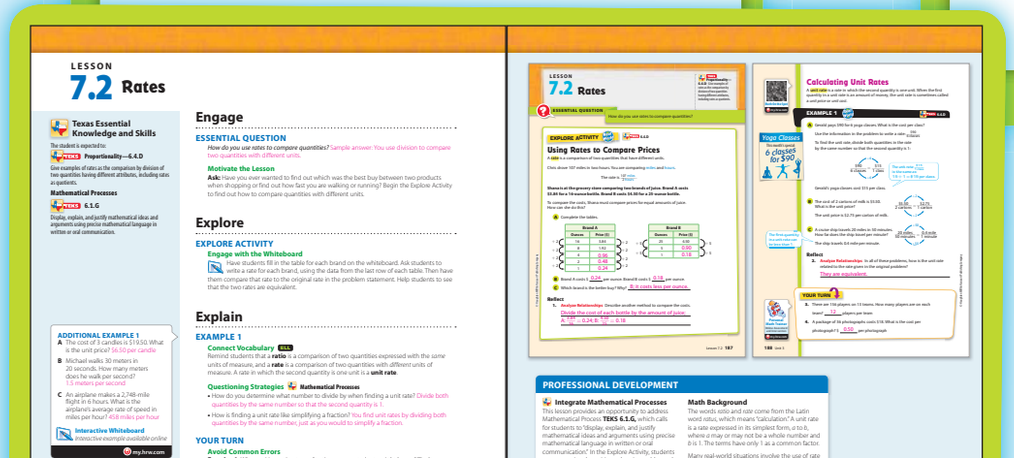
Real-World Videos **Engage students with interesting and relevant applications of the mathematical content of each module.**

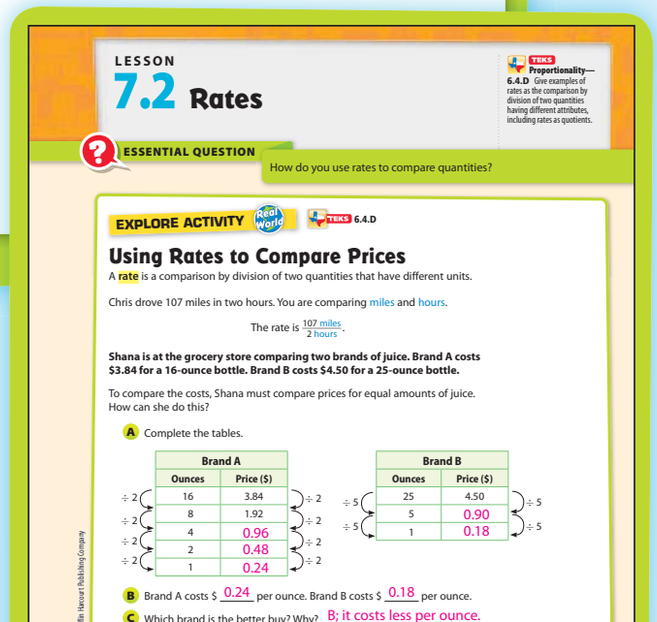
Animated Math **Online interactive simulations, tools, and games help students actively learn and practice key concepts.**



Explore Activities

Students interactively explore new concepts using a variety of tools and approaches.

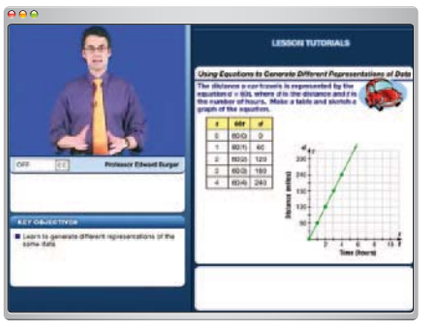




RtI Response to Intervention

Teach

Math on the Spot video tutorials, featuring program authors Dr. Edward Burger and Martha Sandoval-Martinez, accompany every example in the textbook and give students step-by-step instructions and explanations of key math concepts.



Present engaging content on a multitude of devices, including tablets and interactive whiteboards.

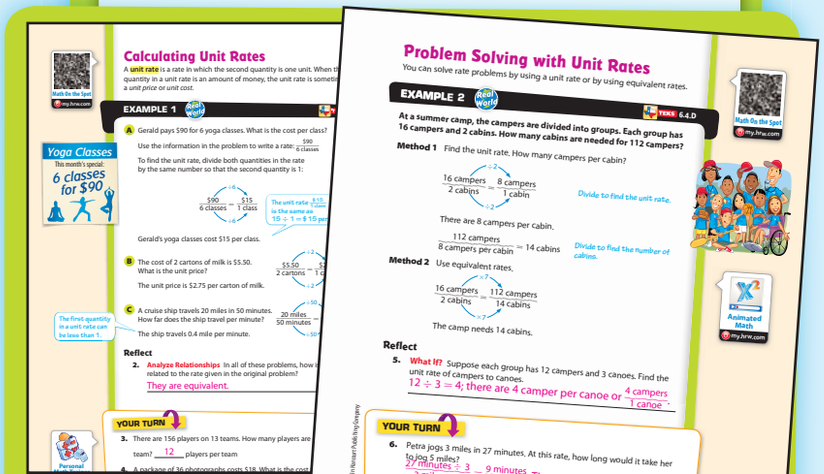
Math Talk

Continually monitor and assess student progress with integrated formative assessment.

Differentiated Instruction Print Resources

Support all learners with Differentiated Instruction Resources, including

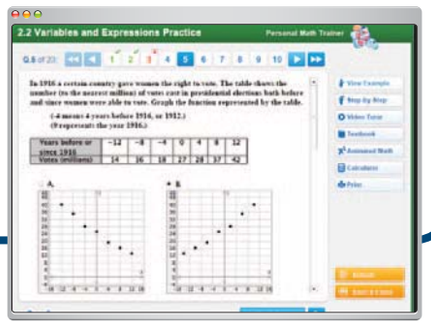
- Leveled Practice and Problem Solving
- Reteach
- Reading Strategies
- Success for English Learners
- Challenge



Assessment and Intervention

The **Personal Math Trainer** provides online practice, homework, assessments, and intervention. Monitor student progress through reports and alerts. Create and customize assignments aligned to specific lessons or TEKS.

- **Practice** – With dynamic items and assignments, students get unlimited practice on key concepts supported by guided examples, step-by-step solutions, and video tutorials.
- **Assessments** – Choose from course assignments or customize your own based on course content, TEKS, difficulty levels, and more.
- **Homework** – Students can complete online homework with a wide variety of problem types, including the ability to enter expressions, equations, and graphs. Let the system automatically grade homework, so you can focus where your students need help the most!
- **Intervention** – Let the Personal Math Trainer automatically prescribe a targeted, personalized intervention path for your students.



H.O.T.

Raise the bar with homework and practice that incorporates higher-order thinking and mathematical processes in every lesson.

Texas Test Prep

Prepare students with practice similar to the Texas assessment program at every module and unit.

Assessment Resources

Tailor assessments to meet the needs of all your classes and students, including

- Leveled Module Quizzes
- Leveled Unit Tests
- Unit Performance Tasks
- Placement, Diagnostic, and Quarterly Benchmark Tests

Math Background

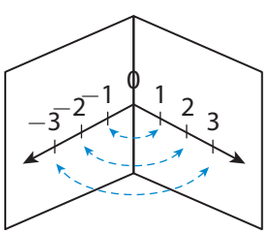
Opposites and Integers TEKS 6.2.B

LESSON 1.1

On a number line, *opposites* are the same distance from 0 but on different sides of 0. For example, 3 and -3 are opposites. Zero is its own opposite. For any real number a, its opposite is written $-a$.

The opposite of a number is also called its additive inverse. This is because the sum of any real number and its opposite is 0. That is, for any real number a, $a + (-a) = 0$. This property is known as the Additive Inverse Property.

The *integers* consist of the whole numbers, $\{0, 1, 2, 3, \ldots\}$, and their opposites, $\{-1, -2, -3, \ldots\}$. Informally, integers can be defined as the real numbers that can be written without a decimal or fractional component. Students should become adept at visualizing the location of the integers on a number line. In particular, they should be aware of their symmetry about 0. If the number line is folded on itself at 0, each integer is paired with its opposite.



Absolute Value TEKS 6.2.B

LESSONS 1.3 and 2.2

The absolute value of a real number is its distance from 0 on a number line. Because distance is always nonnegative, the absolute value of any number is nonnegative. Absolute value can also be defined as follows.

$$|a| = \begin{cases} a, a \geq 0 \\ a, a < 0 \end{cases}$$

This definition states that the absolute value of a nonnegative number is the number itself and that the absolute value of a negative number is the number's opposite. Loosely speaking, this means that the absolute value of a number can be thought of as "the number without its sign." That is, taking the absolute value of a negative number simply removes the minus sign.

The following properties of absolute values should seem intuitively reasonable.

- $|a| = 0$ if and only if $a = 0$
- $|ab| = |a||b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, for $b \neq 0$
- $|-a| = |a|$

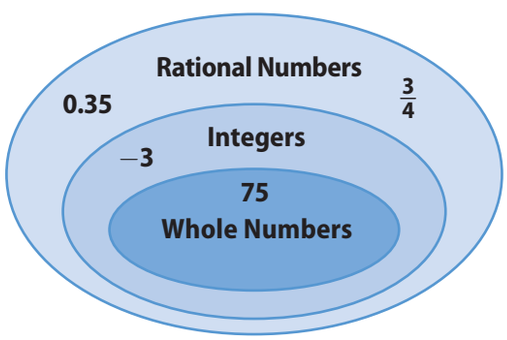
Classifying Rational Numbers TEKS 6.2.A, 6.2.E

LESSON 2.1

A rational number can be written as a quotient of two integers, where the divisor is not zero. To show that a number is rational, rewrite the number as an equivalent ratio of two integers. A whole number such as 5 can be rewritten as $\frac{5}{1}$. Negative rational numbers can be written three ways: $-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$.

A decimal like 1.5 that ends, or terminates, is called a *terminating decimal*. If the same block of digits in a decimal repeats without end, like 0.3333…, the decimal is a *repeating decimal*. Both repeating and terminating decimals are rational numbers.

Rational numbers include integers and whole numbers as shown in the Venn diagram.



The set of integers includes negative integers, zero, and positive integers. Positive numbers are also known as the counting numbers or *natural numbers*. *Whole numbers* are all nonnegative integers, meaning the natural numbers and zero.

In Grade 8, students will learn that rational numbers are part of a larger set of numbers called the *real numbers*. Real numbers include the rational numbers and the irrational numbers. Numbers like π and $\sqrt{2}$ are irrational because they cannot be written as quotient of two integers. When an irrational number is written as a decimal, the digits after the decimal point never terminate and have no repeating pattern.

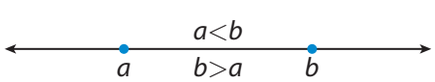
Comparing and Ordering Rational Numbers TEKS 6.2.C, 6.2.D

LESSON 2.3

According to the Law of Trichotomy, given any two rational numbers a and b, exactly one of the following relationships must be true:

- $a < b$
- $a > b$
- $a = b$

Students can visualize the relationships between any two rational numbers by plotting them on a number line. The essential idea is that the value of the numbers increases as you move to the right along the number line. So, if a number a is less than the number b ($a < b$), then a is to the left of b on a number line. This representation also makes clear the equivalent statement that b is greater than a ($b > a$).



UNIT 1

Numbers

MODULE 1
Integers
TEKS 6.2.B, 6.2.C

MODULE 2
Rational Numbers
TEKS 6.2.A, 6.2.B, 6.2.D, 6.2.E

CAREERS IN MATH

Climatologist A climatologist is a scientist who studies long-term trends in climate conditions. These scientists collect, evaluate, and interpret data and use mathematical models to study the dynamics of weather patterns and to understand and predict Earth's climate.

If you are interested in a career in climatology, you should study these mathematical subjects:

- Algebra
- Trigonometry
- Probability and Statistics
- Calculus

Research other careers that require the analysis of data and use of mathematical models.

Unit 1 Performance Task

At the end of the unit, check out how **climatologists** use math.

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Careers in Math

Climatologist

Climatology is based on making accurate measurements of various phenomena and creating mathematical models to make predictions. Climatologists analyze data from diverse sources such as ice cores taken from Antarctica or the rings of trees. You will learn more about analyzing tree rings in the Performance Tasks at the end of the unit.

For more information about careers in mathematics as well as various mathematics appreciation topics, visit the American Mathematical Society at www.ams.org

Vocabulary Preview

Integrating the ELPS

Use the puzzle to give students a preview of important concepts in this unit. Students may work individually, in pairs, or in groups.

ELPS c.4.D Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary to enhance comprehension of written text.

Unit Resources

Go online to access all your unit resources.

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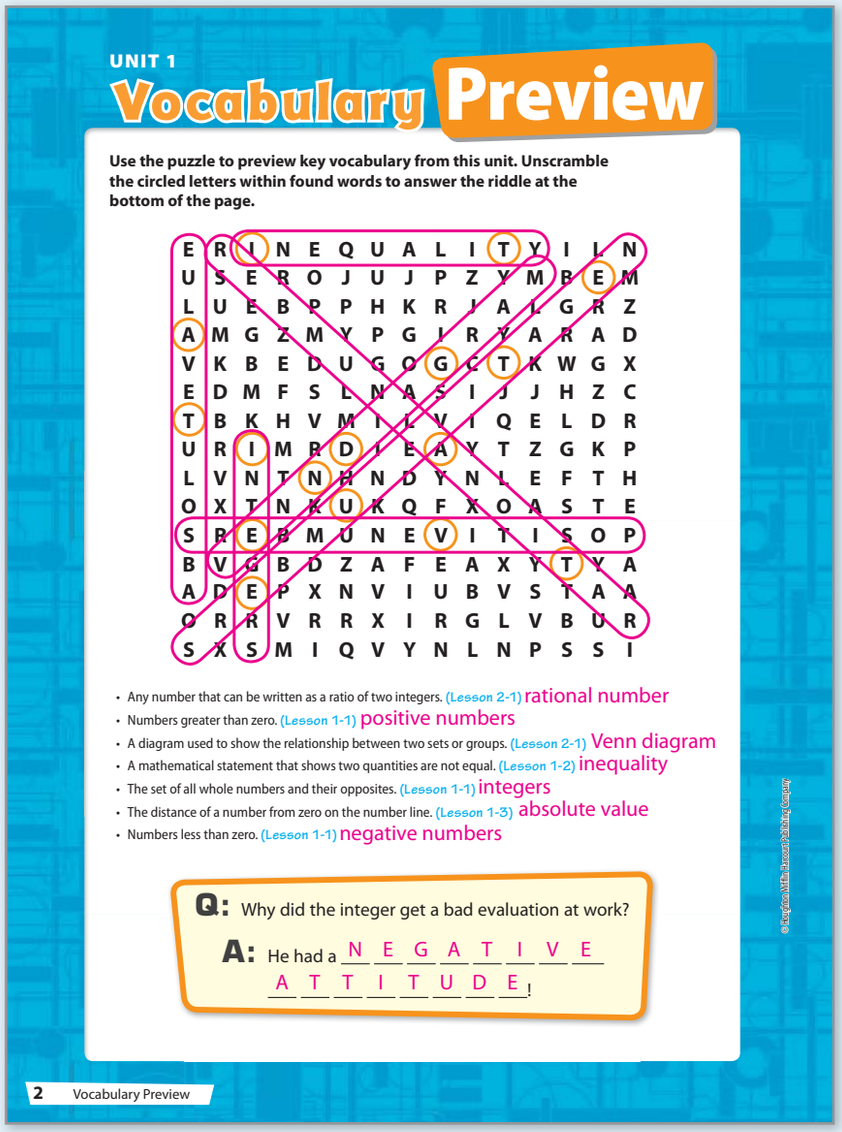
UNIT 1

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

E	R	I	N	E	Q	U	A	L	I	T	Y	I	L	N
U	S	E	R	O	J	U	J	P	Z	Y	M	B	E	M
L	U	E	B	P	P	H	K	R	I	A	L	G	R	Z
A	M	G	Z	M	Y	P	G	I	R	Y	A	R	A	D
V	K	B	E	D	U	G	O	G	C	T	K	W	G	X
E	D	M	F	S	L	N	A	S	I	J	J	H	Z	C
T	B	K	H	V	M	I	L	V	I	Q	E	L	D	R
U	R	I	M	R	D	I	E	A	Y	T	Z	G	K	P
L	V	N	T	N	H	N	D	Y	N	L	E	F	T	H
O	X	T	N	K	U	K	Q	F	X	O	A	S	T	E
S	R	E	B	M	U	N	E	V	I	T	I	S	O	P
B	V	G	B	D	Z	A	F	E	A	X	Y	T	Y	A
A	D	E	P	X	N	V	I	U	B	V	S	T	A	A
O	R	R	V	R	R	X	I	R	G	L	V	B	U	R
S	X	S	M	I	Q	V	Y	N	L	N	P	S	S	I

- Any number that can be written as a ratio of two integers. (Lesson 2-1) rational number
- Numbers greater than zero. (Lesson 1-1) positive numbers
- A diagram used to show the relationship between two sets or groups. (Lesson 2-1) Venn diagram
- A mathematical statement that shows two quantities are not equal. (Lesson 1-2) inequality
- The set of all whole numbers and their opposites. (Lesson 1-1) integers
- The distance of a number from zero on the number line. (Lesson 1-3) absolute value
- Numbers less than zero. (Lesson 1-1) negative numbers

Q: Why did the integer get a bad evaluation at work?

A: He had a N E G A T I V E A T T I T U D E!

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Before	In this Unit	After
Students understand whole numbers, fractions, and decimals: • compare and order • relate fractions and decimals	Students will learn about: • integers and their opposites • absolute value • rational numbers and their opposites • comparing and ordering rational numbers	Students will connect rational numbers and integers: • sets and subsets of rational numbers • perform operations with rational numbers

Integers

ESSENTIAL QUESTION

How can you use integers to solve real-world problems?

You can represent real-world quantities such as temperatures, elevations, and gains and losses of money with positive and negative integers.

MODULE 1

LESSON 1.1
Identifying Integers and Their Opposites
TEKS 6.2.B

LESSON 1.2
Comparing and Ordering Integers
TEKS 6.2.C

LESSON 1.3
Absolute Value
TEKS 6.2.B

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Real-World Video

Integers can be used to describe the value of many things in the real world. The height of a mountain in feet may be a very great integer while the temperature in degrees Celsius at the top of that mountain may be a negative integer.

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Animated Math
Interactively explore key concepts to see how math works.

Personal Math Trainer
Get immediate feedback and help as you work through practice sets.

Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
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Intervention	Enrichment

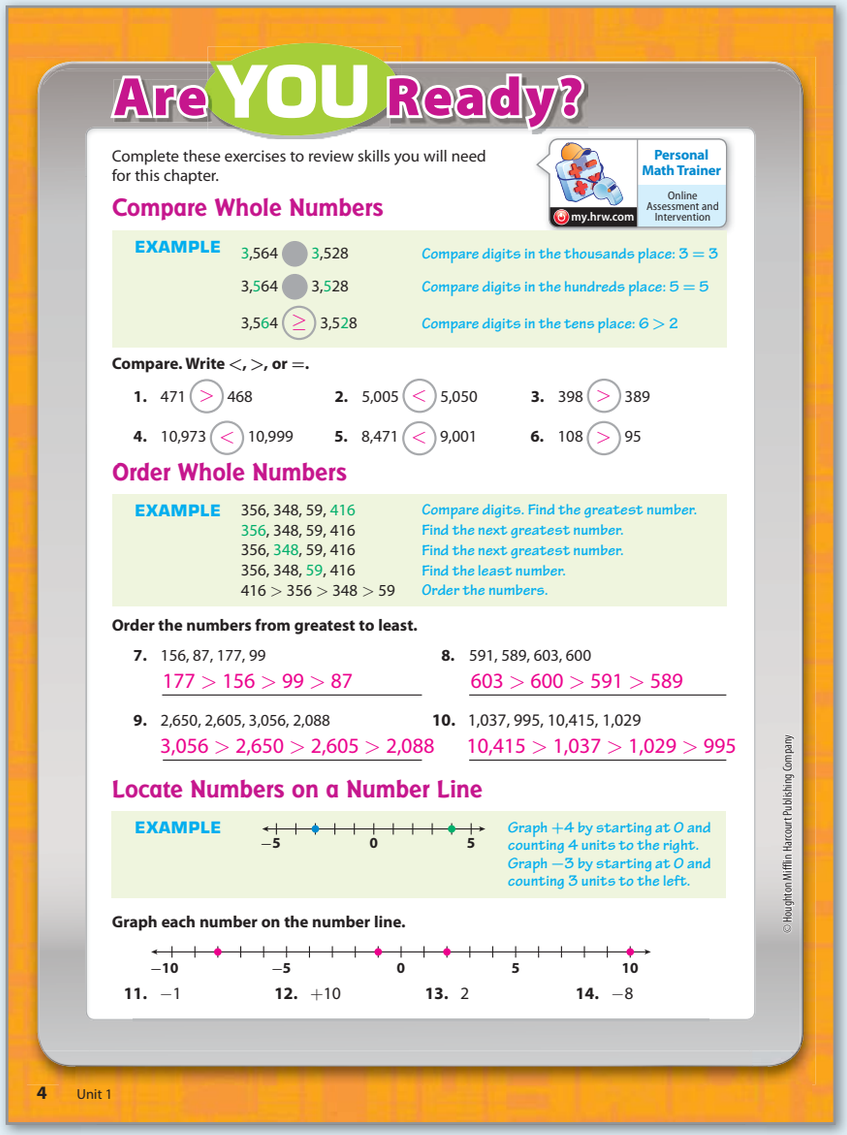
Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 4 Compare Whole Numbers
- Skill 5 Order Whole Numbers
- Skill 61 Locate Numbers on a Number Line

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE



Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.

Personal Math Trainer
Online Assessment and Intervention
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Compare Whole Numbers

EXAMPLE

3,564 ◯ 3,528 — Compare digits in the thousands place: 3 = 3

3,564 ◯ 3,528 — Compare digits in the hundreds place: 5 = 5

3,564 ≥ 3,528 — Compare digits in the tens place: 6 > 2

Compare. Write <, >, or =.

1. 471 > 468
2. 5,005 < 5,050
3. 398 > 389
4. 10,973 < 10,999
5. 8,471 < 9,001
6. 108 > 95

Order Whole Numbers

EXAMPLE

356, 348, 59, 416 — Compare digits. Find the greatest number.
356, 348, 59, 416 — Find the next greatest number.
356, 348, 59, 416 — Find the next greatest number.
356, 348, 59, 416 — Find the least number.
416 > 356 > 348 > 59 — Order the numbers.

Order the numbers from greatest to least.

7. 156, 87, 177, 99
   177 > 156 > 99 > 87
8. 591, 589, 603, 600
   603 > 600 > 591 > 589
9. 2,650, 2,605, 3,056, 2,088
   3,056 > 2,650 > 2,605 > 2,088
10. 1,037, 995, 10,415, 1,029
    10,415 > 1,037 > 1,029 > 995

Locate Numbers on a Number Line

EXAMPLE

Graph +4 by starting at 0 and counting 4 units to the right. Graph −3 by starting at 0 and counting 3 units to the left.

Graph each number on the number line.

11. −1
12. +10
13. 2
14. −8

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PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores integers in an actual sixth-grade classroom.

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Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

ePlanner
Easily plan your classes and access all your resources online.

Interactive Answers and Solutions
Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated, TEKS-aligned practice tests.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The definition and example chart helps students learn the symbols used in this chapter. Explain to students that a symbol is a character that represents a mathematical relationship or operation. To help students understand the concept of symbols, write a few symbols from real life on the board, such the symbol for money or dollar sign ($) and the "at" symbol used in e-mail (@).

Understand Vocabulary

Use the following explanations to help students learn the preview words.

> On a thermometer, if the temperature is above 0, it is written as a **positive number**. If the temperature is below 0, it is written as a **negative number**. For example, if the temperature is 10 degrees below 0, it is written as −10, or minus 10 degrees.

Active Reading

Integrating the ELPS

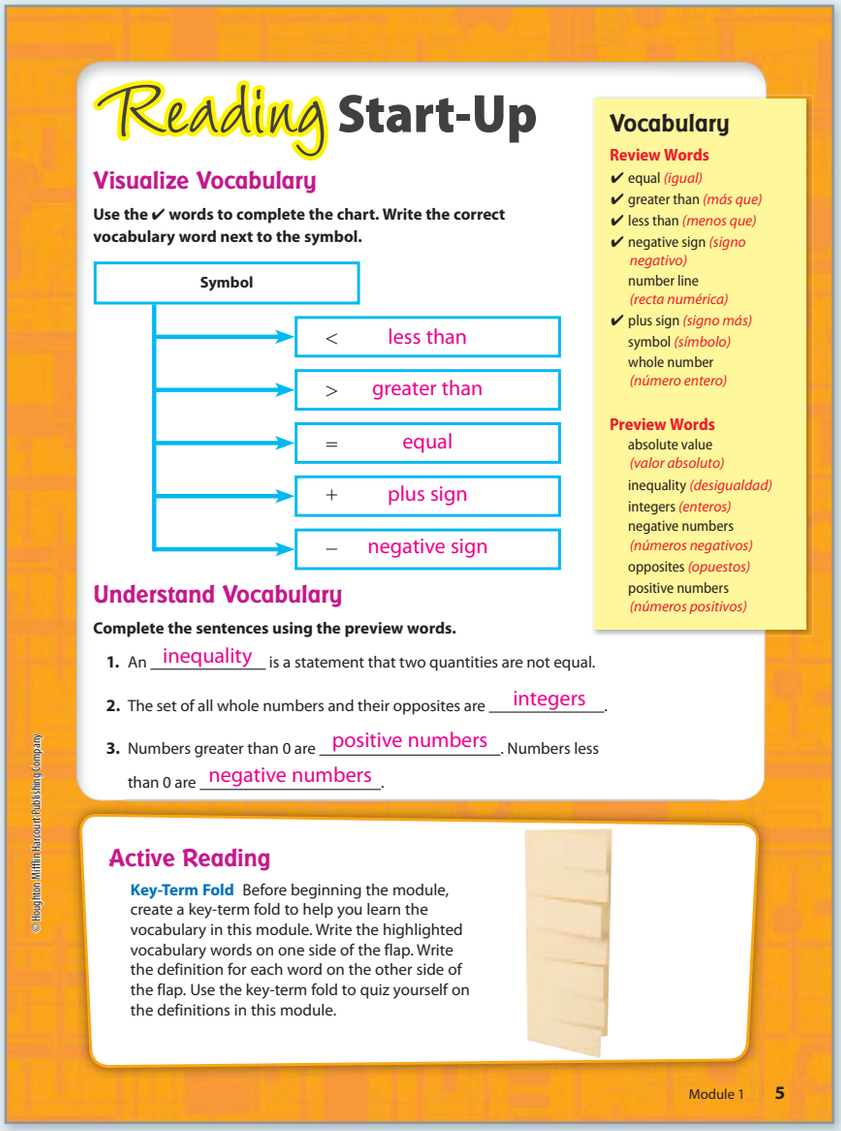
Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

ELPS **c.4.D** Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary to enhance comprehension of written text.

Additional Resources

Differentiated Instruction

- Reading Strategies ELL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the chart. Write the correct vocabulary word next to the symbol.

Symbol	
<	less than
>	greater than
=	equal
+	plus sign
–	negative sign

Understand Vocabulary

Complete the sentences using the preview words.

1. An inequality is a statement that two quantities are not equal.
2. The set of all whole numbers and their opposites are integers.
3. Numbers greater than 0 are positive numbers. Numbers less than 0 are negative numbers.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions in this module.

Vocabulary

Review Words
- ✔ equal *(igual)*
- ✔ greater than *(más que)*
- ✔ less than *(menos que)*
- ✔ negative sign *(signo negativo)*
- number line *(recta numérica)*
- ✔ plus sign *(signo más)*
- symbol *(símbolo)*
- whole number *(número entero)*

Preview Words
- absolute value *(valor absoluto)*
- inequality *(desigualdad)*
- integers *(enteros)*
- negative numbers *(números negativos)*
- opposites *(opuestos)*
- positive numbers *(números positivos)*

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Module 1 5

Grades 6–8 TEKS

Before	In this module	After
Students understand whole numbers, fractions, and decimals: • compare and order whole numbers • compare and order fractions • compare and order decimals	Students recognize, order, and perform computations with integers: • identify a number and its opposite • compare and order integers using a number line • find the absolute value of a number	Students will connect whole numbers and integers: • locate, compare, and order integers using a number line • perform operations with integers

Unpacking the TEKS

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Texas Essential Knowledge and Skills

Content Focal Areas

TEKS Number and Operations—6.2

The student applies mathematical process standards to represent and use rational numbers in a variety of forms.

Integrating the ELPS

ELPS c.4.F Use visual and contextual support . . . to read grade-appropriate content area text . . . and develop vocabulary . . . to comprehend increasingly challenging language.

Go online to see a complete unpacking of the TEKS.

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MODULE 1

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 6.2.B

Identify a number, its opposite, and its absolute value.

Key Vocabulary

Integers *(enteros)*
The set of all whole numbers and their opposites.

opposites *(opuestos)*
Two numbers that are equal distance from zero on a number line.

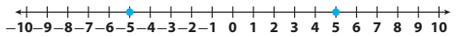
absolute value *(valor absoluto)*
A number's distance from 0 on the number line.

What It Means to You

You will learn see that the absolute value of a number is its distance from 0.

UNPACKING EXAMPLE 6.2.B

Use the number line to determine the absolute values.



$|-5| = 5$ because −5 is 5 units from 0

$|5| = 5$ because 5 is 5 units from 0

TEKS 6.2.C

Locate, compare, and order integers and rational numbers using a number line.

Key Vocabulary

rational number *(número racional)*
Any number that can be expressed as a ratio of two integers.

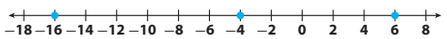
What It Means to You

You can use a number line to order rational numbers.

UNPACKING EXAMPLE 6.2.C

At a golf tournament, David scored +6, Celia scored −16, and Xavier scored −4. One of these three players was the winner of the tournament. Who won the tournament?

The winner will be the player with the lowest score. Draw a number line and graph each player's score.



Celia's score, −16, is the farthest to the left, so it is the lowest score. Celia won the tournament.

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Grade 6 TEKS	Lesson 1.1	Lesson 1.2	Lesson 1.3
TEKS 6.2.B Identify a number, its opposite, and its absolute value.	✓		✓
TEKS 6.2.C Locate, compare, and order integers and rational numbers using a number line.		✓	

LESSON

1.1 Identifying Integers and Their Opposites

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.B

Identify a number, its opposite, and its absolute value.

Mathematical Processes

TEKS 6.1.D

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Engage

ESSENTIAL QUESTION

How do you identify an integer and its opposite? Look for numbers that are the same distance from zero and on opposite sides of zero on the number line; for example, −4 and 4.

Motivate the Lesson

Ask: What is the coldest weather you have ever experienced? Have you ever experienced a temperature that is below zero? How do you write a temperature that is below zero? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1

Focus on Modeling Mathematical Processes

Point out to students that the number line is presented horizontally, but for elevation it is useful to think of it vertically. You may want to draw a vertical number line on the board and label the various locations presented in the table on the vertical number line.

Explain

EXPLORE ACTIVITY 2

Connect Vocabulary ELL ELPS c.1.A

To help students understand the concept of **opposite** in math and in other contexts, make a list with students of pairs of opposites, such as hot and cold, black and white, up and down, left and right. Clarify that left and right is used in the math concept of opposite with negative numbers to the left of 0 and positive numbers to the right. Zero is its own opposite.

Questioning Strategies Mathematical Processes

- Does every integer have an opposite? Explain. Yes, zero is its own opposite. For all other integers, the opposite has a different sign.
- How does a number line help you understand what the opposite of an integer is? I can visually see that 4 and −4 are the same distance from zero.

Connect to Daily Life

Explain that bank statements record amounts of money being withdrawn or spent as negative amounts and amounts of money being deposited as positive amounts.

Talk About It

Check for Understanding

Ask: How do you find the opposite of an integer? Look for the integer that is the same distance from 0 but on the other side of zero.

LESSON 1.1

Identifying Integers and Their Opposites

TEKS Number and operations—6.2.B Identify a number, its opposite, and its absolute value.

ESSENTIAL QUESTION How do you identify an integer and its opposite?

EXPLORE ACTIVITY 1

Real World TEKS 6.2.B

Positive and Negative Numbers

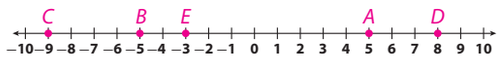
Positive numbers are numbers greater than 0. Positive numbers can be written with or without a plus sign; for example, 3 is the same as +3. **Negative numbers** are numbers less than 0. Negative numbers must always be written with a negative sign.

The number 0 is neither positive nor negative.

The elevation of a location describes its height above or below sea level, which has elevation 0. Elevations below sea level are represented by negative numbers, and elevations above sea level are represented by positive numbers.

A The table shows the elevations of several locations in a state park. Graph the locations on the number line according to their elevations.

Location	Little Butte *A*	Cradle Creek *B*	Dinosaur Valley *C*	Mesa Ridge *D*	Juniper Trail *E*
Elevation (ft)	5	−5	−9	8	−3



B What point on the number line represents sea level? 0

C Which location is closest to sea level? How do you know?

Juniper Trail; its elev. is closest to 0 on the number line.

D Which two locations are the same distance from sea level? Are these locations above or below sea level?

Little Butte (above) and Cradle Creek (below)

E Which location has the least elevation? How do you know?

Dinosaur Valley; its elev. is farthest left on the number line.

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EXPLORE ACTIVITY (cont'd)

Reflect

1. **Analyze Relationships** Morning Glory Stream is 7 feet below sea level. What number represents the elevation of Morning Glory Stream?

   −7

2. **Multiple Representations** Explain how to graph the elevation of Morning Glory Stream on a number line.

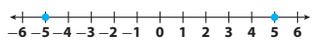
   Graph a point 7 units to the left of 0 on the number line.

EXPLORE ACTIVITY 2

TEKS 6.2.B

Opposites

Two numbers are **opposites** if, on a number line, they are the same distance from 0 but on different sides of 0. For example, 5 and −5 are opposites. 0 is its own opposite.



Integers are the set of all whole numbers and their opposites.

Remember, the set of whole numbers is 0, 1, 2, 3, 4, 5, 6, ...

On graph paper, use a ruler or straightedge to draw a number line. Label the number line with each integer from −10 to 10. Fold your number line in half so that the crease goes through 0. Numbers that line up after folding the number line are opposites.

A Use your number line to find the opposites of 7, −6, 1, and 9. −7; 6; −1; −9

B How does your number line show that 0 is its own opposite?

The crease goes through 0, so 0 lines up with itself.

C What is the opposite of the opposite of 3? 3

Reflect

3. **Justify Reasoning** Explain how your number line shows that 8 and −8 are opposites.

   8 and −8 are the same distance from 0 but on different sides of 0.

4. **Multiple Representations** Explain how to use your number line to find the opposite of the opposite of −6.

   Fold the number line in half at 0. −6 lines up with 6 so 6 is the opposite of −6 and −6 is the opposite of 6. So −6 is the opposite of the opposite of −6.

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.D,** which calls for students to "communicate mathematical ideas … using multiple representations, including symbols, … graphs, and language … as appropriate." In each Explore Activity and Example, students use number lines to represent the integers and opposites that are described with language and/or numbers with or without negative symbols. In this way, students are able to make the connections between and become fluent in using the different representations of integers and their opposites.

Math Background

The opposite of any positive number is negative, and the opposite of any negative number is positive. The sum of a number and its opposite is zero, which is neither positive nor negative.

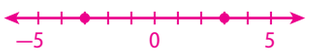
An integer's distance from zero is said to be non-negative instead of positive. When a distance measurement includes a negative symbol, the symbol describes the direction rather than the distance.

ADDITIONAL EXAMPLE 1

The county water department monitors the depth of the reservoir water level each month. The table shows the variation from the optimal depth for four months.

Reservoir Depth Variation from Optimal				
Month	June	July	August	September
Variation (ft)	5	3	−4	−6

A Graph the depth variation for July and its opposite on a number line. What do the numbers represent in this situation?



3 represents positive 3 ft, so in July the water level in the reservoir is 3 ft above the optimal depth. −3 represents 3 ft below the optimal depth.

B The value for October is the opposite of the opposite of the value from August. What was the depth variation in October? −4 ft

Interactive Whiteboard
Interactive example available online

my.hrw.com

EXAMPLE 1

Questioning Strategies Mathematical Processes

- Is the opposite of a temperature always colder? Explain. No, because if the temperature is negative, say −5°, then the opposite would be 5°, which would be warmer.
- Is the opposite of an opposite always the number you started with? Give an example. Yes. If you start at 3, the opposite is −3, then the opposite of −3 is 3.

Engage with the Whiteboard

Have students take turns graphing an integer and then have another student graph the integer's opposite on the number line.

Focus on Patterns Mathematical Processes

Elicit from students that when finding the opposite of the opposite of a positive number, the pattern of the signs in the steps is +, −, +. When finding the opposite of the opposite of a negative number, the pattern of the signs in the steps is −, +, −.

YOUR TURN

Avoid Common Errors

If students seem to get lost with the notation "the opposite of the opposite of," suggest that they work backward through the sentence. First they find the opposite of 6, which is −6. Then they find the opposite of −6.

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you find the opposite of an integer? The opposite of an integer is the integer the same distance from zero on the other side of 0. If the integer is 5, then the opposite is −5. If the integer is −3, then the opposite is 3.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–4, you may want to have students take turns graphing an integer and then have another student graph the integer's opposite on the number lines.

Avoid Common Errors

Exercise 1 Remind students to label the points they graph on the number line carefully, so it is clear which point they intend as the answer.

Exercise 9 Remind students that zero is its own opposite.

Talk About It

Check for Understanding

Ask: I am thinking of a number. The opposite of my number is a distance of 8 units from 0. Do you know what my number is? No, because both 8 and −8 are a distance of 8 units from 0. It could be either 8 or −8.

Integers and Opposites on a Number Line

Positive and negative numbers can be used to represent real-world quantities. For example, 3 can represent a temperature that is 3 °F above 0. −3 can represent a temperature that is 3 °F below 0. Both 3 and −3 are 3 units from 0.

Math On the Spot
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EXAMPLE 1

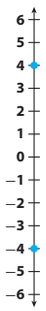
Real World

TEKS 6.2.B

Sandy kept track of the weekly low temperature in her town for several weeks. The table shows the low temperature in °F for each week.

Week	Week 1	Week 2	Week 3	Week 4
Temperature (°F)	−1	3	−4	2

A Graph the temperature from Week 3 and its opposite on a number line. What do the numbers represent?



STEP 1 Graph the value from Week 3 on the number line.

The value from Week 3 is −4.

Graph a point 4 units below 0.

STEP 2 Graph the opposite of −4.

Graph a point 4 units above 0.

The opposite of −4 is 4.

−4 represents a temperature that is 4 °F below 0 and 4 represents a temperature that is 4 °F above 0.

B The value for Week 5 is the opposite of the opposite of the value from Week 1. What was the high temperature in Week 5?

STEP 1 Graph the value from Week 1 on the number line.

The value from Week 1 is −1.

STEP 2 Graph the opposite of −1.

The opposite of −1 is 1.

STEP 3 Graph the opposite of 1.

The opposite of 1 is −1.

The opposite of the opposite of −1 is −1.
The high temperature in Week 5 was −1 °F.

Reflect

5. **Analyze Relationships** Explain how you can find the opposite of the opposite of any number without using a number line.

The opposite of the opposite of a number is the number itself.

My Notes

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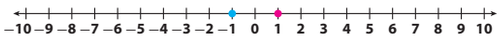
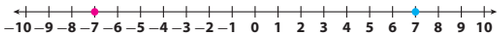
YOUR TURN

Graph the opposite of the number shown on each number line.

6.



7.



Write the opposite of each number.

8. 10 −10 **9.** −5 5 **10.** 0 0

11. What is the opposite of the opposite of 6? 6

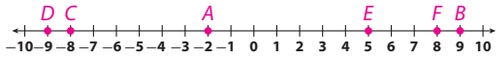
Math Talk
Mathematical Processes
Explain how you could use a number line to find the opposite of 8.

Math Talk anno: First graph a point 8 units to the right of 0. Then grap a point the same distance to the left of 0. That point will be at −8.

Guided Practice

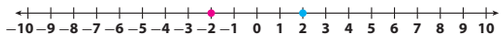
1. Graph and label the following points on the number line. (Explore Activity 1)

a. −2 **b.** 9 **c.** −8 **d.** −9 **e.** 5 **f.** 8

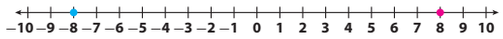


Graph the opposite of the number shown on each number line. (Explore Activity 2 and Example 1)

2.



3.



4.

Write the opposite of each number. (Explore Activity 2 and Example 1)

5. 4 −4 **6.** −11 11 **7.** 3 −3

8. −3 3 **9.** 0 0 **10.** 22 −22

ESSENTIAL QUESTION CHECK-IN

11. Given an integer, how do you find its opposite?

Find the integer that is the same distance from 0 but on the other side of 0.

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DIFFERENTIATE INSTRUCTION

World History

The concept of negative numbers can be traced to Hindu mathematicians. They used negative numbers to represent debts, as we do today, and formulated rules for the arithmetic of integers. Their ideas were acquired by Arab mathematicians, who passed the ideas on to European scientists over time.

Manipulatives

For Explore Activity 2, some students have difficulty labeling a number line and folding it so the opposite integers line up. It may be helpful to give them printed number lines with a vertical dashed line through zero.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

1.1 LESSON QUIZ

TEKS 6.2.B

Sara keeps a record of the money that she deposits and withdraws from her account each week.

Week	1	2	3
Account entry ($)	$4	$10	−$8

1. Which week(s) does Sara have a negative entry in her account?
2. Graph each value and its opposite on a number line.
3. Which week's entry was the closest to zero?
4. For Week 4, Sara's entry is the opposite of the opposite of her entry on Week 1. What is her Week 4 entry?

Lesson Quiz available online

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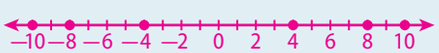
Answers

1. Week 3
2. 

3. Week 1
4. $4

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS 6.2.B

Concepts & Skills	Practice
Explore Activity 1 Positive and Negative Numbers	Exercises 1, 12, 23, 24
Explore Activity 2 Opposites	Exercises 2–10, 12, 13, 15, 18, 19–24
Example 1 Integers and Opposites on a Number Line	Exercises 2–10, 14, 16, 17, 20–23

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
12	**2** Skills/Concepts	**1.A** Everyday life
13–18	**1** Recall of Information	**1.C** Select tools
19–23	**2** Skills/Concepts	**1.C** Select tools
24	**3** Strategic Thinking H.O.T.	**1.A** Everyday life
25	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments
26	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
27	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments
28	**3** Strategic Thinking H.O.T.	**1.C** Select tools

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

1.1 Independent Practice

TEKS 6.2.B

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12. **Chemistry** Atoms normally have an electrical charge of 0. Certain conditions, such as static, can cause atoms to have a positive or a negative charge. Atoms with a positive or negative charge are called *ions*.

Ion	A	B	C	D	E
Charge	−3	+1	−2	+3	−1

a. Which ions have a negative charge?

A, C, E

b. Which ions have charges that are opposites?

A and D; B and E

c. Which ion's charge is not the opposite of another ion's charge?

C

Name the integer that meets the given description.

13. the opposite of −17 17

14. 4 units left of 0 −4

15. the opposite of the opposite of 2 2

16. 15 units right of 0 15

17. 12 units right of 0 12

18. the opposite of −19 19

19. **Analyze Relationships** Several wrestlers are trying to lose weight for a competition. Their change in weight since last week is shown in the chart.

Wrestler	Tino	Victor	Ramsey	Baxter	Luis
Weight Change (in pounds)	−2	6	2	5	−5

a. Did Victor lose or gain weight since last week? gain

b. Which wrestler's weight change is the opposite of Ramsey's? Tino

c. Which wrestlers have lost weight since last week? Tino and Luis

d. Frankie's weight change since last week was the opposite of Victor's.

What was Frankie's weight change? −6

e. Frankie's goal last week was to gain weight. Did he meet his goal? Explain.

No; −6 pound change means Frankie lost 6 pounds.

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Find the distance between the given number and its opposite on a number line.

20. 6 12 units

21. −2 4 units

22. 0 0 units

23. −7 14 units

24. **What If?** Three contestants are competing on a trivia game show. The table shows their scores before the final question.

Contestant	Score Before Final Question
Timothy	−25
Shawna	18
Kaylynn	−14

a. How many points must Shawna earn for her score to be the opposite of Timothy's score before the final question? 7 points

b. Which person's score is closest to 0? Kaylynn

c. Who do you think is winning the game before the final question? Explain.

Shawna; she is the only player with a positive score.

H.O.T. FOCUS ON HIGHER ORDER THINKING

25. **Communicate Mathematical Ideas** Which number is farther from 0 on a number line: −9 or 6? Explain your reasoning.

−9; it is 9 units away from 0 on a number line, and 6 is only 6 units away from 0.

26. **Analyze Relationships** A number is k units to the left of 0 on the number line. Describe the location of its opposite.

Its opposite is k units to the right of 0 on the number line.

27. **Critique Reasoning** Roberto says that the opposite of a certain integer is −5. Cindy concludes that the opposite of an integer is always negative. Explain Cindy's error.

Cindy assumed the original integer is always positive. But if the original integer is negative, its opposite will be positive.

28. **Multiple Representations** Explain how to use a number line to find the opposites of the integers 3 units away from −7.

10, 4; −10 is 3 units to the left of −7 and 10 is the opposite of −10. −4 is 3 units to the right of −7 and 4 is the opposite of −4.

Work Area

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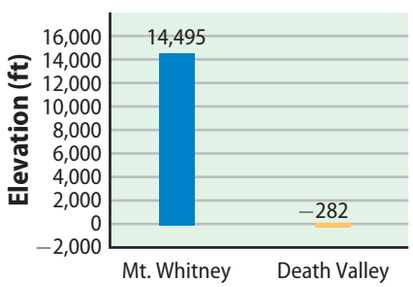
EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The lowest and highest places in the United States are both in California, as shown in the graph. How can you use the graph to find the difference in elevation between the two locations?

If you start at the lowest point, you need to go up 282 ft to sea level and then another 14,495 ft to get to the top of Mt. Whitney. 282 + 14,495 = 14,777.



LESSON

1.2 Comparing and Ordering Integers

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.C

Locate, compare, and order integers and rational numbers using a number line.

Mathematical Processes

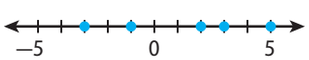
TEKS 6.1.C

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

ADDITIONAL EXAMPLE 1

Tia's golf scores during her first five days at a golf academy are shown in the table. Graph the scores on a number line, and then list the numbers in order from least to greatest.

Day	Mon	Tue	Wed	Thu	Fri
Score	5	−1	3	2	−3



−3, −1, 2, 3, 5

Interactive Whiteboard
Interactive example available online

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Engage

ESSENTIAL QUESTION

How do you compare and order integers? Graph the integers on a number line, and then read the integers in order from left to right to order them from least to greatest.

Motivate the Lesson

Ask: Which temperature is colder: −20° or −8°? How can you decide? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Focus on Reasoning

Point out to students that teams with negative win/loss records have more losses than wins, while those with positive records have more wins than losses. So when comparing records, if there are more negative than positive records, the league is not very successful, and conversely, if there are more positive then negative records, the league is successful.

Explain

EXAMPLE 1

Talk About It

Check for Understanding

Ask: How does a number line help you order a set of integers? A number line provides a visual representation of the values of the integers in order from least to greatest from left to right.

Questioning Strategies

Mathematical Processes

- What is the best score Fred recorded for the week and when does it occur? How do you know? −5 and it occurs on Thursday. −5 has the least value of all the scores recorded and in golf the lowest score, not the highest score, wins the game.
- How do integers change as you move farther left from zero on the number line? They decrease in value.

YOUR TURN

Engage with the Whiteboard

Have students take turns graphing the numbers on the number line and then have another student list the numbers in order from least to greatest.

Talk About It

Check for Understanding

Ask: What do the following changes in stock prices mean: −$5, $4, and $0? A change of −$5 means the stock price fell $5, a change of $4 means the stock price rose $4, and a change of $0 means the stock price did not change.

LESSON 1.2

Comparing and Ordering Integers

TEKS Number and operations—6.2.C Locate, compare, and order integers ... using a number line.

ESSENTIAL QUESTION How do you compare and order integers?

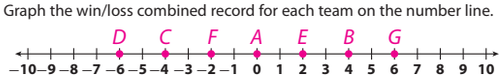
EXPLORE ACTIVITY Real World TEKS 6.2.C

Comparing Positive and Negative Integers

The Westfield soccer league ranks its teams using a number called the "win/loss combined record." A team with more wins than losses will have a positive combined record, and a team with fewer wins than losses will have a negative combined record. The table shows the total win/loss combined record for each team at the end of the season.

Team	Sharks *A*	Jaguars *B*	Badgers *C*	Tigers *D*	Cougars *E*	Hawks *F*	Wolves *G*
Win/Loss Combined Record	0	4	−4	−6	2	−2	6

A Graph the win/loss combined record for each team on the number line.



B Which team had the best record in the league? How do you know?

Wolves; their record is farthest right on the number line.

C Which team had the worst record? How do you know?

Tigers; their record is farthest left on the number line.

Reflect

1. **Analyze Relationships** Explain what the data tell you about the win/loss records of the teams in the league.

The Badgers, Tigers, and Hawks lost more games than they won. The Jaguars, Cougars, and Wolves won more games than they lost. The Sharks won the same number of games as they lost.

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Ordering Positive and Negative Integers

When you read a number line from left to right, the numbers are in order from least to greatest.

EXAMPLE 1 Real World TEKS 6.2.C

Fred recorded the following golf scores during his first week at the golf academy. In golf, the player with the lowest score wins the game.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Score	4	−2	3	−5	−1	0	−3

Graph Fred's scores on the number line, and then list the numbers in order from least to greatest.

Math Talk Mathematical Processes
What day did Fred have his best golf score? How do you know?

Thursday; Fred got his lowest score, −5, on Thursday, and in golf the lowest score is the best.

STEP 1 Graph the scores on the number line.

STEP 2 Read from left to right to list the scores in order from least to greatest.

The scores listed from least to greatest are −5, −3, −2, −1, 0, 3, 4.

YOUR TURN

Graph the values in each table on a number line. Then list the numbers in order from greatest to least.

2.

Change in Stock Price ($)					
−5	4	0	−3	−6	2

4, 2, 0, −3, −5, −6

3.

Elevation (meters)							
9	−1	−6	2	−10	0	5	8

9, 8, 5, 2, 0, −1, −6, −10

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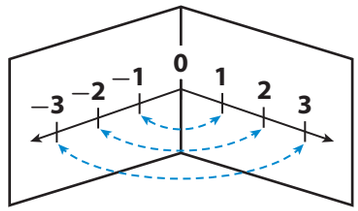
PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.C,** which calls for students to "select tools … and techniques, including … number sense as appropriate, to solve problems." In the Explore Activity and in both Examples, students use a number line to order and compare integers in real-world contexts, such as ordering rankings, golf scores, and comparing annual precipitation. In this way, students are able to see the integers in terms of their relationship to zero, to each other, and to create statements of numerical order in real-world contexts.

Math Background

The integers consist of the whole numbers, {0, 1, 2, 3, …}, and their opposites, {0, −1, −2, −3, …}. Informally, integers can be defined as the real numbers that can be written without a decimal or fractional component. Students should become adept at visualizing the location of the integers on a number line. In particular, they should be aware of their symmetry about 0. If the number line is folded on itself at 0, each integer is paired with its opposite.



ADDITIONAL EXAMPLE 2

In 1989, many cities in Texas experienced record low temperatures. The Dallas/Ft. Worth area had a record low of −1 °F, and San Angelo had a record low of −4 °F. Which of the two cities had the colder record low temperature in 1989? Write an inequality to support your answer.

San Angelo; $-4 < -1$

Interactive Whiteboard

Interactive example available online

my.hrw.com

EXAMPLE 2

Questioning Strategies

Mathematical Processes

- Can you always write two different inequality statements to compare two numbers with different values? Explain. Yes, because you can use $>$ to compare the larger number to the smaller number and $<$ to compare the smaller number to the larger number.
- If −1 is the greatest negative integer, is there a least negative integer? Explain. No, the set of negative numbers is infinite, so every negative integer on the number line has an integer of lesser value to its left.

Avoid Common Errors

If students have trouble in determining which inequality sign to use, you may want to remind them that the inequality sign always points to the lesser of two numbers.

YOUR TURN

Avoid Common Errors

When students work with negative numbers, they often think that the number with the greater absolute value is the greater number. You may want to remind them that for negative numbers, the number with the greater absolute value is actually the lesser number because it is farther away from zero in the negative direction.

Elaborate

Talk About It

Summarize the Lesson

Ask: How is a number line used to compare and order integers? When the numbers are graphed they are in order of their value. The number line shows the numbers from least to greatest (left to right) and from greatest to least (right to left).

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2, have students use the number line given in Exercise 1 to graph and order the integers.

Avoid Common Errors

Exercise 1 Remind students that the coldest temperature is the least temperature, the one farthest to the left on the number line.

Exercises 2–3 Caution students to pay attention to the signs of the numbers when they create their ordered lists.

Exercise 8 Remind students that when comparing negative integers, the number with the greater absolute value is actually the lesser number because values decrease as one moves left from zero.

Writing Inequalities

An **inequality** is a statement that two quantities are not equal. The symbols $<$ and $>$ are used to write inequalities.

- The symbol $>$ means "is greater than."
- The symbol $<$ means "is less than."

You can use a number line to help write an inequality.

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EXAMPLE 2

Real World

TEKS 6.2.C

A In 2005, Austin, Texas, received 51 inches in annual precipitation. In 2009, the city received 36 inches in annual precipitation. In which year was there more precipitation?

Graph 51 and 36 on the number line.

- 51 is to the *right* of 36 on the number line.

  This means that 51 is **greater than** 36.

  Write the inequality as $51 > 36$.

- 36 is to the *left* of 51 on the number line.

  This means that 36 is **less than** 51.

  Write the inequality as $36 < 51$.

  There was more precipitation in 2005.

No; for any positive integer, you can find greater positive integers to the right of it on the number line; yes, -1.

B Write two inequalities to compare -6 and 7.
$-6 < 7$; $7 > -6$

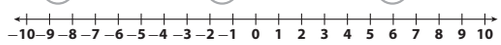
C Write two inequalities to compare -9 and -4.
$-4 > -9$; $-9 < -4$

Math Talk
Mathematical Processes
Is there a greatest integer? Is there a greatest negative integer? Explain.

YOUR TURN

Compare. Write $>$ or $<$. Use the number line to help you.

4. -10 ($<$) -2 **5.** -6 ($<$) 6 **6.** -7 ($>$) -8



7. Write two inequalities to compare -2 and -18. $-2 > -18$; $-18 < -2$

8. Write two inequalities to compare 39 and -39. $-39 < 39$; $39 > -39$

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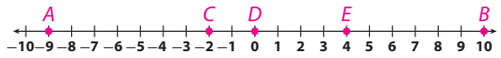
Guided Practice

1a. Graph the temperature for each city on the number line. (Explore Activity)

City	A	B	C	D	E
Temperature (°F)	−9	10	−2	0	4



b. Which city was coldest? A

c. Which city was warmest? B

List the numbers in order from least to greatest. (Example 1)

2. 4, −6, 0, 8, −9, 1, −3
$-9, -6, -3, 0, 1, 4, 8$

3. −65, 34, 7, −13, 55, 62, −7
$-65, -13, -7, 7, 34, 55, 62$

4. Write two inequalities to compare −17 and −22. $-17 > -22$; $-22 < -17$

Compare. Write $<$ or $>$. (Example 2)

5. -9 ($<$) 2 **6.** 0 ($<$) 6 **7.** 3 ($>$) -7 **8.** 5 ($>$) -10

9. -1 ($>$) -3 **10.** -8 ($<$) -4 **11.** -4 ($<$) 1 **12.** -2 ($>$) -6

13. Compare the temperatures for the following cities. Write $<$ or $>$. (Example 2)

City	Alexandria	Redwood Falls	Grand Marais	Winona	International Falls
Average Temperature in March (°C)	−3	0	−2	2	−4

a. Alexandria and Winona $-3 < 2$

b. Redwood Falls and International Falls $0 > -4$

ESSENTIAL QUESTION CHECK-IN

14. How can you use a number line to compare and order numbers?
The numbers on a number line are in order from least to greatest as you move from left to right.

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DIFFERENTIATE INSTRUCTION

Kinesthetic Experience

Have students write the integers being compared on sticky notes and arrange them on a large number line on the board. Ask them to explain why they placed the numbers in the position they did, and encourage them to rearrange the notes if placed incorrectly. Then have students write two inequalities for each comparison they make.

Number Sense

Have students practice comparing numbers without a number line by visualizing them on a number line. For example, **Ask:** *Would −125 be to the left or to the right of −76 on a number line?* Have students challenge one another to tell whether a number is located to the left or right of another number on the number line.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online Assessment and Intervention

Online homework assignment available

my.hrw.com

1.2 LESSON QUIZ

TEKS **6.2.C**

Use a number line to list the numbers in order from least to greatest.

1. 4, −1, 6, 0, −4, 5, −3

2. 9, −12, −5, 8, −15, 0, −2

3. −38, −16, 45, −24, 71, −63, 10

4. Write two inequalities to compare −13 and −26.

5. Write two inequalities to compare 0 and −8.

6. At the end of a golf game, Jared's score was −3 and Ned's score was −5. Who won the game? Explain your reasoning.

Lesson Quiz available online

my.hrw.com

Answers

1. −4, −3, −1, 0, 4, 5, 6

2. −15, −12, −5, −2, 0, 8, 9

3. −63, −38, −24, −16, 10, 45, 71

4. −13 > −26; −26 < −13

5. 0 > −8; −8 < 0

6. Ned; In golf, the player with the lowest score wins.

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS **6.2.C**

Concepts & Skills	Practice
Explore Activity Comparing Positive and Negative Integers	Exercises 1, 15, 19
Example 1 Ordering Positive and Negative Integers	Exercises 2–4, 18, 19
Example 2 Writing Inequalities	Exercises 5–13, 16, 17, 20–23

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
15	**2** Skills/Concepts	**1.D** Multiple representations
16–18	**2** Skills/Concepts	**1.A** Everyday life
19	**2** Skills/Concepts	**1.E** Create and use representations
20–23	**2** Skills/Concepts	**1.A** Everyday life
24	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
25	**3** Strategic Thinking H.O.T.	**1.A** Everyday life
26	**3** Strategic Thinking H.O.T.	**1.A** Everyday life
27	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

1.2 Independent Practice

TEKS 6.2.C

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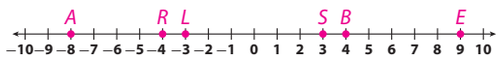
15. Multiple Representations A hockey league tracks the plus-minus records for each player. A plus-minus record is the difference in even strength goals for and against the team when a player is on the ice. The following table lists the plus-minus values for several hockey players.

Player	A. Jones	B. Sutter	E. Simpson	L. Mays	R. Tomas	S. Klatt
Plus-minus	−8	4	9	−3	−4	3

a. Graph the values on the number line.



b. Which player has the best plus-minus record? E. Simpson

Astronomy The table lists the average surface temperature of some planets. Write an inequality to compare the temperatures of each pair of planets.

Planet	Average Surface Temperature (°C)
Mercury	167
Uranus	−197
Neptune	−200
Earth	15
Mars	−65
Jupiter	−110

16. Uranus and Jupiter −197 < −110

17. Mercury and Mars 167 > −65

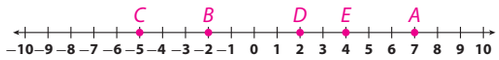
18. Arrange the planets in order of average surface temperature from greatest to least. Mercury, Earth, Mars, Jupiter, Uranus, Neptune

19. Represent Real-World Problems For a stock market project, five students each invested pretend money in one stock. They tracked gains and losses in the value of that stock for one week. In the following table, a gain is represented by a positive number and a loss is represented by a negative number.

Students	Andre	Bria	Carla	Daniel	Ethan
Gains and Losses ($)	7	−2	−5	2	4

Graph the students' results on the number line. Then list them in order from least to greatest.

a. Graph the values on the number line.



b. The results listed from least to greatest are −5, −2, 2, 4, 7.

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Geography The table lists the lowest elevation for several countries. A negative number means the elevation is below sea level, and a positive number means the elevation is above sea level. Compare the lowest elevation for each pair of countries. Write < or >.

Country	Lowest Elevation (feet)
Argentina	−344
Australia	−49
Czech Republic	377
Hungary	249
United States	−281

20. Argentina and the United States −344 < −281

21. Czech Republic and Hungary 377 > 249

22. Hungary and Argentina 249 > −344

23. Which country in the table has the lowest elevation? Argentina

24. Analyze Relationships There are three numbers a, b, and c, where $a > b$ and $b > c$. Describe the positions of the numbers on a number line.

The first number, a, will be the farthest to the right on the number line. The third number, c, will be farthest to the left on the number line. The second number, b, will be between a and c on the number line.

H.O.T. FOCUS ON HIGHER ORDER THINKING

25. Critique Reasoning At 9 A.M. the outside temperature was −3°F. By noon, the temperature was −12°F. Jorge said that it was getting warmer outside. Is he correct? Explain.

No; −12°F < −3°F, so it was getting colder outside.

26. Problem Solving Golf scores represent the number of strokes above or below par. A negative score means that you hit a number below par while a positive score means that you hit a number above par. The winner in golf has the lowest score. During a round of golf, Angela's score was −5 and Lisa's score was −8. Who won the game? Explain.

Lisa won the game because she had the lowest score.

27. Look for a Pattern Order −3, 5, 16, and −10 from least to greatest. Then order the same numbers from closest to zero to farthest from zero. Describe how your lists are similar. Would this be true if the numbers were −3, 5, −16 and −10?

−10, −3, 5, 16 and −3, 5, −10, 16; both lists end with 16 because 16 is the greatest number and is farthest from zero. This would not be true for the second group of numbers because in that list, the least number, −16, would be farthest from zero.

Work Area

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EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity In a game on a number line, the starting line is at zero. Each player makes three consecutive jumps. A forward jump is represented by a positive number, and a backward jump is represented by a negative number.

1st jump: The player makes a jump away from the starting line and lands on a point.

2nd jump: From the point where he or she lands, the player makes a second jump towards the starting line.

3rd jump: Now the player makes a third jump away from the starting line.

Rachel makes three consecutive jumps of 8 feet, −5 feet, and 6 feet.

Andy makes three consecutive jumps of −10 feet, 7 feet, and −4 feet.

Who is closest to the starting line at the end of the round? Explain. You may find it helpful to use a counter and a number line to track each player's jumps.

Rachel jumps forward 8 ft, then back 5 ft, and then forward 6 ft. She is now 9 ft in front of the starting line. Andy jumps back 10 ft, then forward 7 ft, and then back 4 ft. He is now 7 ft behind the starting line. Since −7 is closer to zero than 9, Andy is closest to the starting line at the end of the round.

LESSON

1.3 Absolute Value

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.B

Identify a number, its opposite, and its absolute value.

Mathematical Processes

TEKS 6.1.A

Apply mathematics to problems arising in everyday life, society, and the workplace.

ADDITIONAL EXAMPLE 1

A deep-sea diver dived off a boat to a depth of −45 feet. What is the absolute value that expresses the distance the diver went? The absolute value of −45 is 45.

Interactive Whiteboard
Interactive example available online

my.hrw.com

Animated Math
Absolute Values and Opposites

Students explore integers, their opposites, and their absolute values with a dynamic number line.

my.hrw.com

Engage

ESSENTIAL QUESTION

How do you find and use absolute value? Count the distance from zero to a number on a number line. Absolute value is always nonnegative and is useful for representing distance or an amount of change.

Motivate the Lesson

Ask: Have you ever borrowed money from a friend? How can you mathematically describe owing money? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1

Connect to Daily Life

Point out to students that they can use absolute value to describe or compare real-life distances such as how far they ride a bike, dive under water, or ascend in a Ferris wheel.

Explain

EXAMPLE 1

Focus on Communication

Discuss with students why an absolute value of $25 may be used to describe a −$25 change to the balance of a gift card.

Engage with the Whiteboard

Have students take turns graphing a number on a number line and showing how to use the number line to find the absolute value of the number.

Questioning Strategies Mathematical Processes

- How would you define a balance on a gift card? It is the amount of money that is available to the cardholder.
- How would you explain what a balance of $0.00 on a gift card means? It means that the card has no monetary value.
- How can you use absolute value to show the amount Jake has left on his gift card? Find the absolute value of each item Jake bought. Add the two values to find the absolute value of his purchases. Subtract that number from the balance on his gift card.

LESSON

1.3 Absolute Value

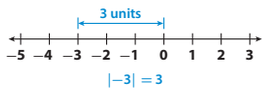
TEKS Number and operations—6.2.B Identify a number, its opposite, and its absolute value.

ESSENTIAL QUESTION How do you find and use absolute value?

EXPLORE ACTIVITY 1 TEKS 6.2.B

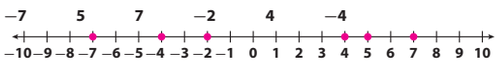
Finding Absolute Value

The absolute value of a number is the number's distance from 0 on a number line. For example, the absolute value of −3 is 3 because −3 is 3 units from 0. The absolute value of −3 is written $|-3|$.



Because absolute value represents a distance, it is always nonnegative.

Graph the following numbers on the number line. Then use your number line to find each absolute value.



A $|-7| =$ 7 B $|5| =$ 5 C $|7| =$ 7

D $|-2| =$ 2 E $|4| =$ 4 F $|-4| =$ 4

Reflect

1. **Analyze Relationships** Which pairs of numbers have the same absolute value? How are these numbers related?

   −7 and 7; 4 and −4; they are opposites.

2. **Justify Reasoning** Negative numbers are less than positive numbers. Does this mean that the absolute value of a negative number must be less than the absolute value of a positive number? Explain.

   No; $-7 < 3$ but $|-7| > |3|$; the distance from −7 to 0 is greater than the distance from 3 to 0.

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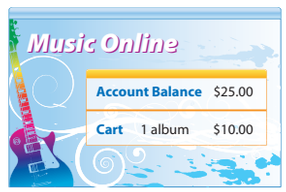
Absolute Value In A Real-World Situation

In real-world situations, absolute values are often used instead of negative numbers. For example, if you use a $50 gift card to make a $25 purchase, the change in your gift card balance can be represented by −$25.

EXAMPLE 1 Real World TEKS 6.2.B

Jake uses his online music store gift card to buy an album of songs by his favorite band.

Find the negative number that represents the change in the balance on Jake's card after his purchase. Explain how absolute value would be used to express that number in this situation.



Math Talk Mathematical Processes
Explain why the price Jake paid for the album is represented by a negative number.

Jake pays $10 so there will be a change of −$10 in Jake's gift card balance.

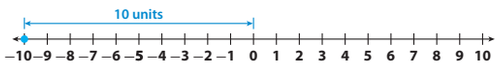
STEP 1 Find the negative integer that represents the change in the balance.

−$10 *The balance decreased by $10, so use a negative number.*

The balance went down, so the change is a negative number.

STEP 2 Use the number line to find the absolute value of −$10.

−10 is 10 units from 0 on the number line.



The absolute value of −$10 is $10, or $|-10| = 10$.

The balance on Jake's card decreased by $10.

Reflect

3. **Communicate Mathematical Ideas** Explain why the absolute value of a number will never be negative.

   The absolute value of a number is its distance from 0 on the number line. Distance can never be negative.

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.A,** which calls for students to "apply mathematics to problems arising in everyday life, society, and the workplace." Example 1 and Explore Activity 2 draw direct connections between absolute value and real-world situations, including the amount owed on a credit card and the amount of money stored on a gift card.

Math Background

You can interpret absolute value as the magnitude of a real number without regard to its sign. It measures the amount of change rather than the direction of change; the farther a number is from 0, the greater its absolute value. This is easy to visualize on a number line. You can also look at it mathematically:

$|n| = n$ if n is ≥ 0

$|n| = -n$ if n is < 0

YOUR TURN

Avoid Common Errors

Make sure that students understand that the absolute value of any negative integer is its *distance* from zero on a number line, which is always expressed as a *nonnegative* number.

Talk About It

Check for Understanding

Ask: What can you say about the distance of numbers −55 and 55 from 0? Because they are opposites, they are both the same distance from 0 and have the same absolute value.

EXPLORE ACTIVITY 2

Connect Vocabulary ELL

Point out to students that when working with money, a loss or a debt can be represented by a negative number. So, in Explore Activity 2, the negative amounts represent money that you spent, a negative change.

Talk About It

Check for Understanding

Ask: How can you tell which person owes the most money? His or her balance will have the greatest absolute value.

Questioning Strategies Mathematical Processes

- If a person has a credit card balance of $50 and has a −$30 change in their balance, how do you find the amount the person owes? Find the absolute value of −$30, which is $30, and add it to $50. The person now owes $80.
- If a person's credit card balance decreases, what happens to the amount the person owes? It decreases.
- When a person makes a payment on their credit card, what happens to the amount of money available on the card (card limit) and to the amount the person owes (card balance)? The amount of money available (card limit) will increase while the amount the person owes (card balance) will decrease.

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you use absolute value to compare two negative numbers, such as fees, or amounts owed on a credit card or other kind of loan? You compare the absolute values of the negative numbers; the negative number with the greater absolute value is the lesser amount, indicating a greater amount owed.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2, have students count the tick marks to show that the distance from 0 to −10 is 10.

Avoid Common Errors

Exercise 2 If students have difficulty understanding how a credit or a fee affects the bill, remind them that a credit is like a payment, it will decrease the balance, while a fee is like a purchase, it will increase the balance.

YOUR TURN

4. The temperature at night reached −13 °F. Write an equivalent statement about the temperature using the absolute value of the number.

The temperature at night reached 13 °F below zero.

Find each absolute value.

5. $|-12|$ 12 **6.** $|91|$ 91 **7.** $|-55|$ 55

8. $|0|$ 0 **9.** $|88|$ 88 **10.** $|1|$ 1

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EXPLORE ACTIVITY 2 Real World TEKS 6.2.B

Comparing Absolute Values

You can use absolute values to compare negative numbers in real-world situations.

Maria, Susan, George, and Antonio checked their credit card balances on their smartphones. The amounts owed are shown.

Susan George Antonio Maria

Answer the following questions. When you have finished, you will have enough clues to match each statement with the correct person.

Remember: When someone owes a positive amount of money, this means that he or she has a *negative* balance.

A Maria's credit card balance is less than −$30. Does Maria owe more than $30 or less than $30? more than $30

B Susan's credit card balance is greater than −$25. Does Susan owe more than $25 or less than $25? less than $25

C George's credit card balance is $5 less than Susan's balance. Does George owe more than Susan or less than Susan? more than Susan

D Antonio owes $15 less than Maria owes. This means that Antonio's balance is greater than Maria's balance.

E Write each person's name underneath his or her smartphone.

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EXPLORE ACTIVITY 2 (cont'd)

Reflect

11. **Analyze Relationships** Use absolute value to describe the relationship between a negative credit card balance and the amount owed.

The amount owed is the absolute value of the balance.

Guided Practice

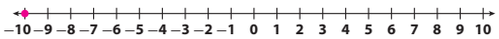
1. **Vocabulary** If a number is negative, then the number is less than its absolute value. (Explore Activity 1)

2. If Ryan pays his car insurance for the year in full, he will get a credit of $28. If he chooses to pay a monthly premium, he will pay a $10 late fee for any month that the payment is late. (Explore Activity 1, Example 1)

a. Which of these values could be represented with a negative number? Explain.

−$10; it is a fee, so it represents a change of −$10 in the amount of money Ryan has.

b. Use the number line to find the absolute value of your answer from part a. $|-10| = 10$



3. Leo, Gabrielle, Sinea, and Tomas are playing a video game. Their scores are described in the table below. (Explore Activity 2)

Name	Leo	Gabrielle	Sinea
Score	less than −100 points	20 more points than Leo	50 points less than Leo

a. Leo wants to earn enough points to have a positive score. Does he need to earn more than 100 points or less than 100 points? more than 100

b. Gabrielle wants to earn enough points to not have a negative score. Does she need to earn more points than Leo or less points than Leo? less than Leo

c. Sinea wants to earn enough points to have a higher score than Leo. Does she need to earn more than 50 points or less than 50 points? more than 50

ESSENTIAL QUESTION CHECK-IN

4. When is the absolute value of a number equal to the number?

when the number is nonnegative

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DIFFERENTIATE INSTRUCTION

Home Connection

Students may be unfamiliar with how loans work. Discuss that many people borrow money they need to buy expensive items like cars, furniture, computers, and homes. Discuss that people pay back the money they borrow over a period of time and they pay fees for that privilege. Invite students to cite some examples with which they are familiar. Then have them define absolute value in their own words and then explain how it is used to express the amount of money borrowed.

Critical Thinking

Ask: How does the relationship between a negative number and its absolute value compare with the relationship between a nonnegative number and its absolute value?

A nonnegative number is *equal to* its absolute value; a negative number is *less than* its absolute value.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

1.3 LESSON QUIZ

TEKS **6.2.B**

1. Mia's credit card balance is less than −$85. Does she owe more or less than $85?
2. Leon has a gift card for $100. He spent $65 of it on books. Describe the change in Leon's card balance in two different ways.
3. The record low temperature in Oregon is −54 °F. Use absolute value to express that temperature in degrees below zero.
4. Nick's bank account balance changed by $34 one month and by −$82 the next month. Which amount represents the lesser change?

Lesson Quiz available online

my.hrw.com

Answers

1. She owes more than $85.
2. Use the negative number −$65 to represent the change in the value of Leon's card; use absolute value to say that his balance will be $65 less.
3. 54 degrees below zero
4. $34

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS **6.2.B**

Concepts & Skills	Practice
Explore Activity 1 Finding Absolute Value	Exercises 1, 2
Example 1 Absolute Value in a Real-World Situation	Exercises 2, 5, 8–11
Explore Activity 2 Comparing Absolute Values	Exercises 3, 6, 7

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
5	**2** Skills/Concepts	**1.A** Everyday life
6	**2** Skills/Concepts	**1.F** Analyze relationships
7	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments
8	**2** Skills/Concepts	**1.F** Analyze relationships
9	**2** Skills/Concepts	**1.A** Everyday life
10	**2** Skills/Concepts	**1.A** Everyday life
11	**2** Skills/Concepts	**1.A** Everyday life
12	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
13	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
14	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

1.3 Independent Practice

TEKS 6.2.B

Personal Math Trainer
Online Assessment and Intervention

5. **Financial Literacy** Jacob earned $80 babysitting and deposited the money into his savings account. The next week he spent $85 on video games. Use integers to describe the weekly changes in Jacob's savings account balance.

The first week his balance changed by +$80. The second week his balance changed by −$85.

6. **Financial Literacy** Sara's savings account balance changed by $34 one week and by −$67 the next week. Which amount represents the greatest change? −$67

7. **Analyze Relationships** Bertrand collects movie posters. The number of movie posters in his collection changes each month as he buys and sells posters. The table shows how many posters he bought or sold in the given months.

Month	January	February	March	April
Posters	Sold 20	Bought 12	Bought 22	Sold 28

a. Which months have changes that can be represented by positive numbers? Which months have changes that can be represented by negative numbers? Explain.

February and March represent positive numbers because Bertrand bought posters. January and April represent negative numbers because Bertrand sold posters.

b. According to the table, in which month did the size of Bertrand's poster collection change the most? Use absolute value to explain your answer.

April; He sold 28 posters which can be represented by −28. The absolute value of −28 is 28, the greatest of any month.

8. **Earth Science** Death Valley has an elevation of −282 feet relative to sea level. Explain how to use absolute value to describe the elevation of Death Valley as a positive integer.

The absolute value of −282 is 282 so Death Valley is 282 feet below sea level.

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9. **Communicate Mathematical Ideas** Lisa and Alice are playing a game. Each player either receives or has to pay play money based on the result of their spin. The table lists how much a player receives or pays for various spins.

Red	Pay $5
Blue	Receive $4
Yellow	Pay $1
Green	Receive $3
Orange	Pay $2

a. Express the amounts in the table as positive and negative numbers.

−5, 4, −1, 3, −2

b. Describe the change to Lisa's amount of money when the spinner lands on red.

The spinner landing on red results in a change of −$5 to Lisa's amount of money.

10. **Financial Literacy** Sam's credit card balance is less than −$36. Does Sam owe more or less than $36? Sam owes more than $36.

11. **Financial Literacy** Emily spent $55 from her savings on a new dress. Explain how to describe the change in Emily's savings balance in two different ways.

Use a negative integer to say that Emily's balance changed by −$55; Use absolute value to say that Emily's balance is $55 less.

H.O.T. FOCUS ON HIGHER ORDER THINKING

12. **Make a Conjecture** Can two different numbers have the same absolute value? If yes, give an example. If no, explain why not.

Yes, it is possible. For example, $|-1| = 1$ and $|1| = 1$.

13. **Communicate Mathematical Ideas** Does $-|-4| = |-(-4)|$? Justify your answer.

No; $-|-4| = -4$, and $|-(-4)| = |4| = 4$.

14. **Critique Reasoning** Angelique says that finding the absolute value of a number is the same as finding the opposite of the number. For example, $|-5| = 5$. Explain her error.

Angelique's technique only works if the original number is negative. The absolute value of a nonnegative number is equal to the number itself, not its opposite.

Work Area

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EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Read each statement carefully. Write *True* or *False*.

1. $|14| > 14$ False

2. $|44| = |-44|$ True

3. $|-17| = 17$ True

4. $|-22| = -22$ False

5. $-n$ and n have the same absolute value. True

6. $|-33|$ is the opposite of -33. True

7. Rewrite one number in Exercises 1–4 above to make each false statement true and each true statement false.

1. $|14| > -14$ True
2. $-|44| = |-44|$ False
3. $|-17| = -17$ False
4. $|-22| = 22$ True

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

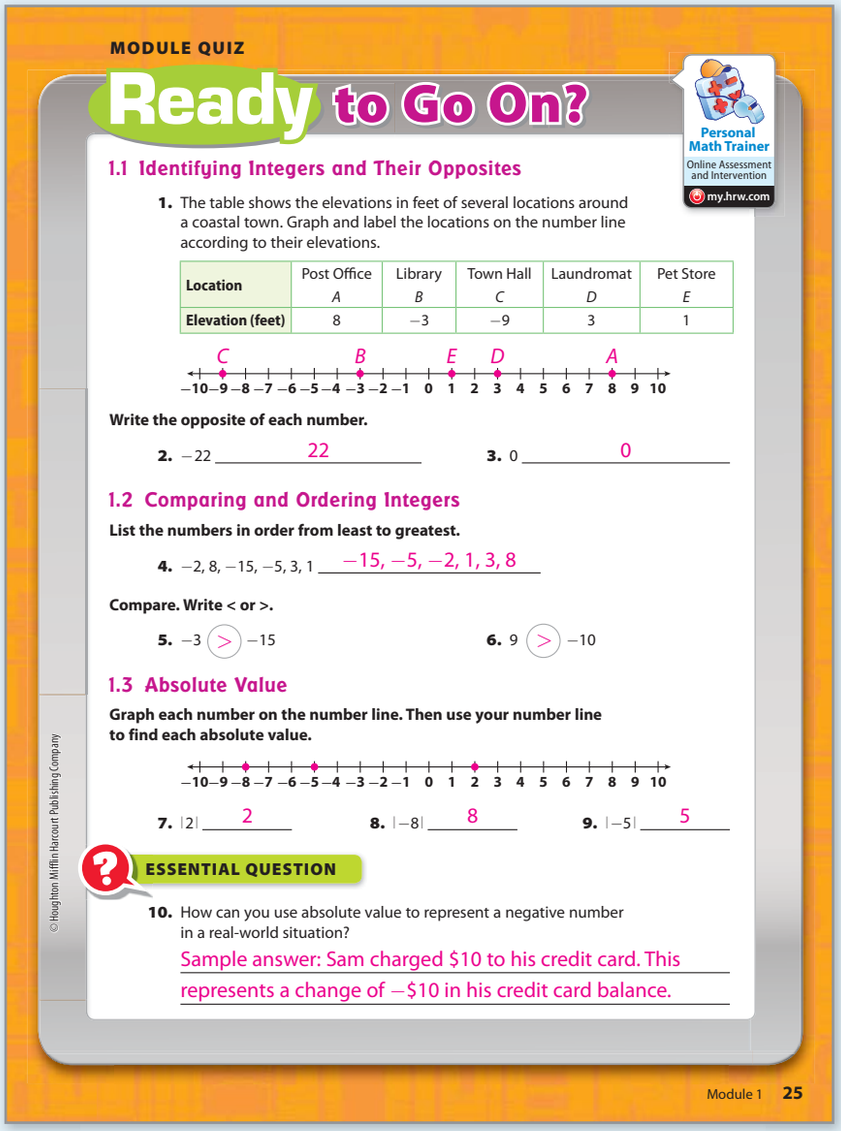
Online and Print Resources

Differentiated Instruction	*Differentiated Instruction*
• Reteach worksheets	• Challenge worksheets PRE-AP
• Reading Strategies ELL	Extend the Math PRE-AP
• Success for English Learners ELL	Lesson Activities in TE

Additional Resources

Assessment Resources includes:

• Leveled Module Quizzes



MODULE QUIZ

Ready to Go On?

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1.1 Identifying Integers and Their Opposites

1. The table shows the elevations in feet of several locations around a coastal town. Graph and label the locations on the number line according to their elevations.

Location	Post Office A	Library B	Town Hall C	Laundromat D	Pet Store E
Elevation (feet)	8	−3	−9	3	1

C B E D A

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

Write the opposite of each number.

2. −22 22 **3.** 0 0

1.2 Comparing and Ordering Integers

List the numbers in order from least to greatest.

4. −2, 8, −15, −5, 3, 1 −15, −5, −2, 1, 3, 8

Compare. Write < or >.

5. −3 > −15 **6.** 9 > −10

1.3 Absolute Value

Graph each number on the number line. Then use your number line to find each absolute value.

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

7. |2| 2 **8.** |−8| 8 **9.** |−5| 5

ESSENTIAL QUESTION

10. How can you use absolute value to represent a negative number in a real-world situation?

Sample answer: Sam charged $10 to his credit card. This represents a change of −$10 in his credit card balance.

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Module 1 25

Texas Essential Knowledge and Skills

Lesson	Exercises	TEKS
1.1	1–3	**6.2.B**
1.2	4–6	**6.2.C**
1.3	7–10	**6.2.B**

Texas Test Prep

Texas Testing Tip Students can draw a diagram, graph, or picture to help organize information from a test item.

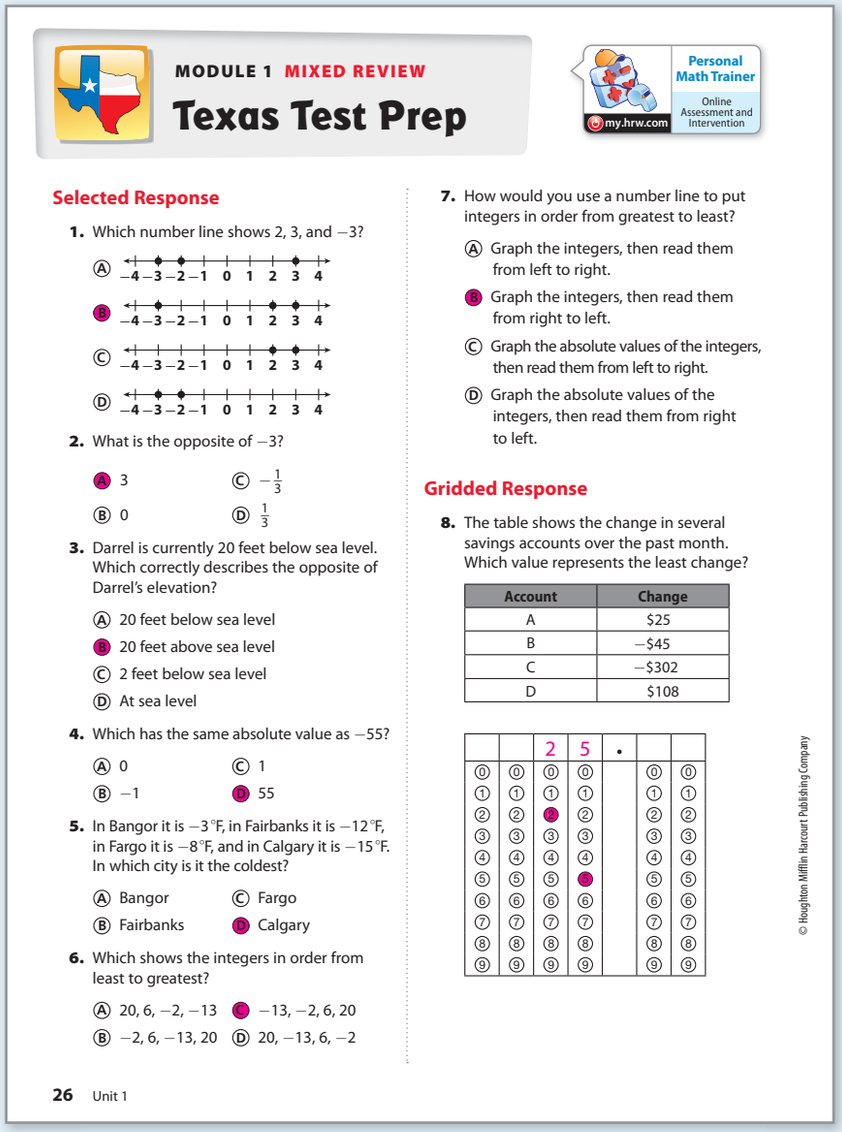
Item 5 If students sketch a number line and plot a point for the temperature of each city, Calgary's point will be the farthest to the left. This means Calgary is the coldest, and therefore the correct answer.

Item 6 If students notice that each answer choice uses the same numbers in a different order, they can sketch a number line and plot the numbers from any of the answer choices. Reading the plotted points from left to right gives the order of the numbers from least to greatest, revealing C as the correct answer.

Avoid Common Errors

Item 2 Students may read the term *opposite* and think that the answer will be negative. Point out that they need to find the opposite of negative 3, which is positive 3.

Item 7 Caution students to read the question carefully so they understand what is being asked. The question asks for the numbers to be ordered from greatest to least rather than from least to greatest.



MODULE 1 MIXED REVIEW

Texas Test Prep

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Selected Response

1. Which number line shows 2, 3, and −3?

Ⓐ −4 −3 −2 −1 0 1 2 3 4

Ⓑ −4 −3 −2 −1 0 1 2 3 4

Ⓒ −4 −3 −2 −1 0 1 2 3 4

Ⓓ −4 −3 −2 −1 0 1 2 3 4

2. What is the opposite of −3?

Ⓐ 3 Ⓒ $-\frac{1}{3}$

Ⓑ 0 Ⓓ $\frac{1}{3}$

3. Darrel is currently 20 feet below sea level. Which correctly describes the opposite of Darrel's elevation?

Ⓐ 20 feet below sea level
Ⓑ 20 feet above sea level
Ⓒ 2 feet below sea level
Ⓓ At sea level

4. Which has the same absolute value as −55?

Ⓐ 0 Ⓒ 1

Ⓑ −1 Ⓓ 55

5. In Bangor it is −3°F, in Fairbanks it is −12°F, in Fargo it is −8°F, and in Calgary it is −15°F. In which city is it the coldest?

Ⓐ Bangor Ⓒ Fargo

Ⓑ Fairbanks Ⓓ Calgary

6. Which shows the integers in order from least to greatest?

Ⓐ 20, 6, −2, −13 Ⓒ −13, −2, 6, 20

Ⓑ −2, 6, −13, 20 Ⓓ 20, −13, 6, −2

7. How would you use a number line to put integers in order from greatest to least?

Ⓐ Graph the integers, then read them from left to right.
Ⓑ Graph the integers, then read them from right to left.
Ⓒ Graph the absolute values of the integers, then read them from left to right.
Ⓓ Graph the absolute values of the integers, then read them from right to left.

Gridded Response

8. The table shows the change in several savings accounts over the past month. Which value represents the least change?

Account	Change
A	$25
B	−$45
C	−$302
D	$108

		2	5	.		
0	0	0	0		0	0
1	1	1	1		1	1
2	2	2	2		2	2
3	3	3	3		3	3
4	4	4	4		4	4
5	5	5	5		5	5
6	6	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	9		9	9

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26 Unit 1

Texas Essential Knowledge and Skills

Items	Grade 6 TEKS	Mathematical Process TEKS
1*	6.2.C	6.1.D
2	6.2.B	6.1.F
3	6.2.B	6.1.A
4	6.2.B	6.1.F
5	6.2.C	6.1.A
6	6.2.C	6.1.E
7	6.2.C	6.1.F, 6.1.G
8	6.2.C	6.1.A

* Item integrates mixed review concepts from previous modules or a previous course.

Rational Numbers

MODULE 2

ESSENTIAL QUESTION

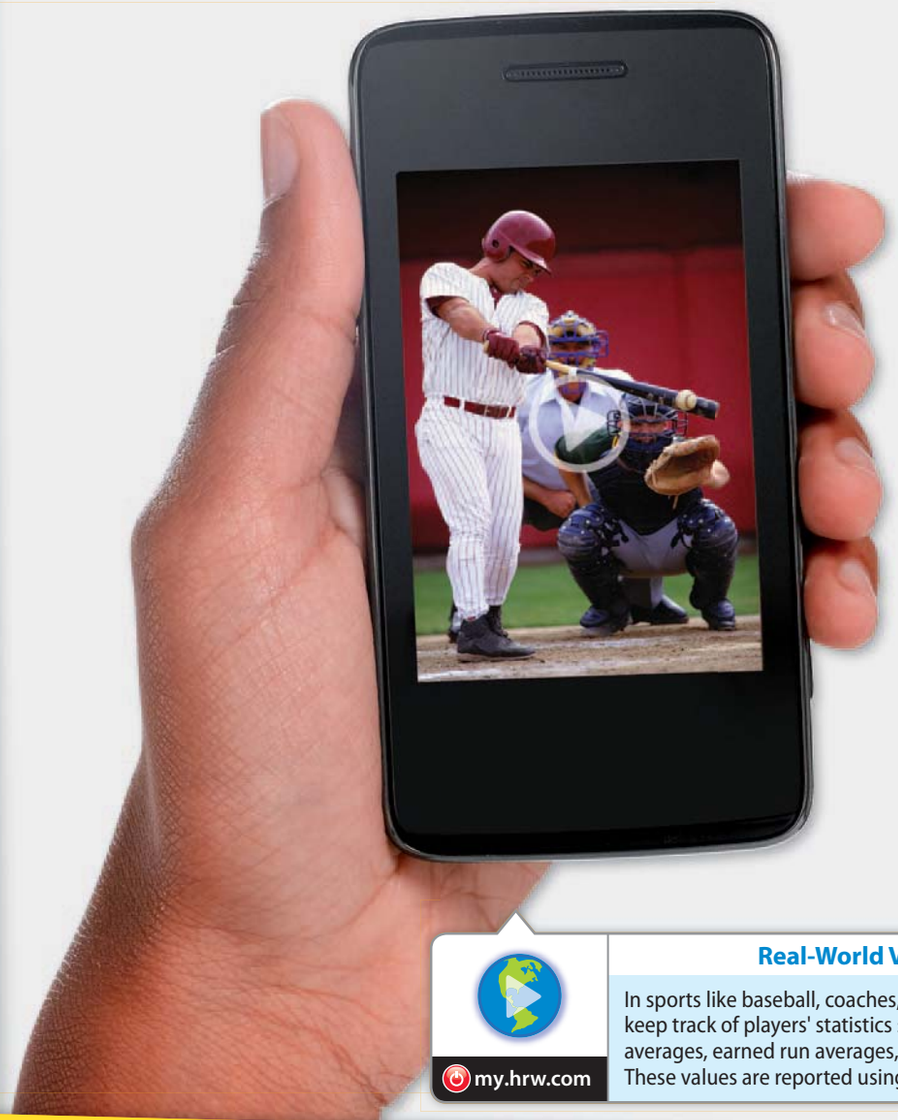
How can you use rational numbers to solve real-world problems?

You can represent any real-world quantity that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, as a rational number.

LESSON 2.1
Classifying Rational Numbers
TEKS 6.2.A, 6.2.E

LESSON 2.2
Identifying Opposites and Absolute Value of Rational Numbers
TEKS 6.2.B

LESSON 2.3
Comparing and Ordering Rational Numbers
TEKS 6.2.D



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Real-World Video

In sports like baseball, coaches, analysts, and fans keep track of players' statistics such as batting averages, earned run averages, and runs batted in. These values are reported using rational numbers.

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Animated Math
Interactively explore key concepts to see how math works.

Personal Math Trainer
Get immediate feedback and help as you work through practice sets.

Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
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Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

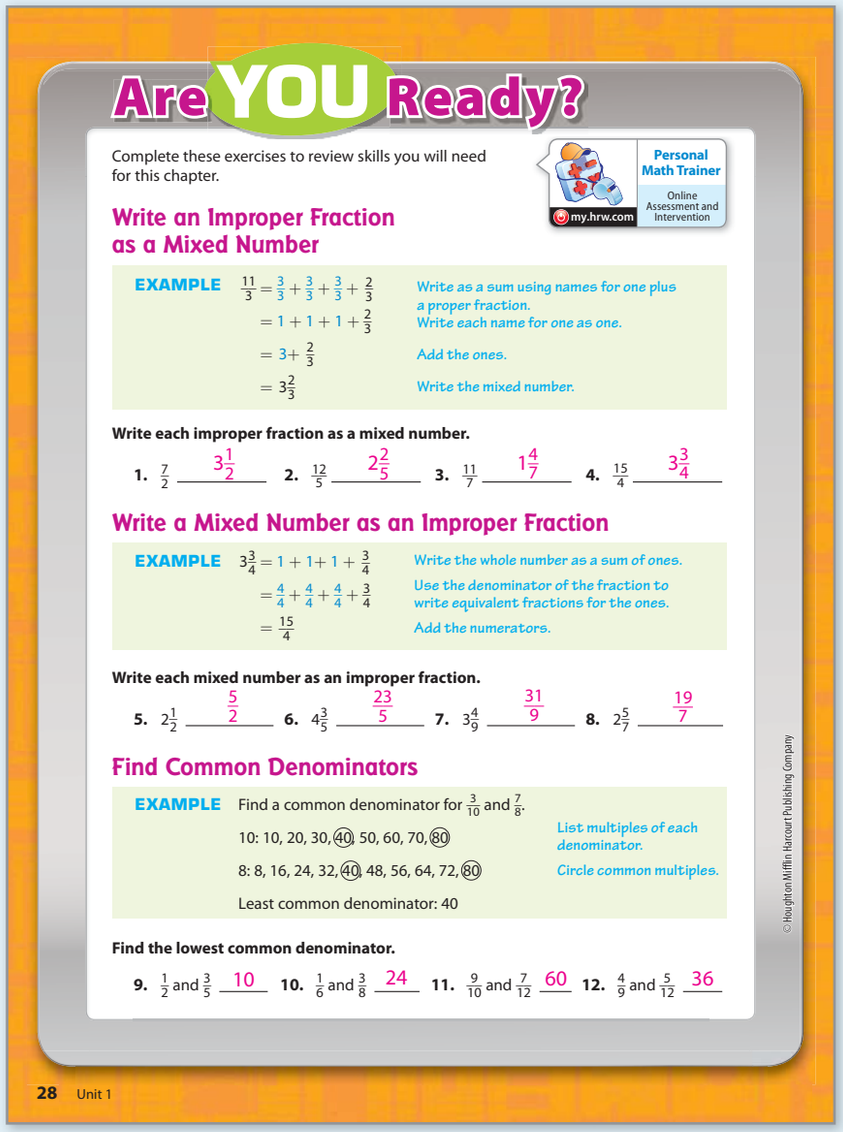
Intervention	Enrichment

Online and Print Resources

Skills Intervention worksheets
- Skill 21 Write an Improper Fraction as a Mixed Number
- Skill 22 Write a Mixed Number as an Improper Fraction
- Skill 23 Find Common Denominators

Differentiated Instruction
- Challenge worksheets **PRE-AP**

Extend the Math **PRE-AP** Lesson Activities in TE



Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.

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Online Assessment and Intervention
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Write an Improper Fraction as a Mixed Number

EXAMPLE

$\frac{11}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$ Write as a sum using names for one plus a proper fraction. Write each name for one as one.

$= 1 + 1 + 1 + \frac{2}{3}$

$= 3 + \frac{2}{3}$ Add the ones.

$= 3\frac{2}{3}$ Write the mixed number.

Write each improper fraction as a mixed number.

1. $\frac{7}{2}$ $3\frac{1}{2}$ 2. $\frac{12}{5}$ $2\frac{2}{5}$ 3. $\frac{11}{7}$ $1\frac{4}{7}$ 4. $\frac{15}{4}$ $3\frac{3}{4}$

Write a Mixed Number as an Improper Fraction

EXAMPLE

$3\frac{3}{4} = 1 + 1 + 1 + \frac{3}{4}$ Write the whole number as a sum of ones.

$= \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$ Use the denominator of the fraction to write equivalent fractions for the ones.

$= \frac{15}{4}$ Add the numerators.

Write each mixed number as an improper fraction.

5. $2\frac{1}{2}$ $\frac{5}{2}$ 6. $4\frac{3}{5}$ $\frac{23}{5}$ 7. $3\frac{4}{9}$ $\frac{31}{9}$ 8. $2\frac{5}{7}$ $\frac{19}{7}$

Find Common Denominators

EXAMPLE Find a common denominator for $\frac{3}{10}$ and $\frac{7}{8}$.

10: 10, 20, 30, (40), 50, 60, 70, (80) List multiples of each denominator.

8: 8, 16, 24, 32, (40), 48, 56, 64, 72, (80) Circle common multiples.

Least common denominator: 40

Find the lowest common denominator.

9. $\frac{1}{2}$ and $\frac{3}{5}$ 10 10. $\frac{1}{6}$ and $\frac{3}{8}$ 24 11. $\frac{9}{10}$ and $\frac{7}{12}$ 60 12. $\frac{4}{9}$ and $\frac{5}{12}$ 36

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28 Unit 1

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores rational numbers in an actual sixth-grade classroom.

Professional Development
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GO DIGITAL
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Online Teacher Edition
Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

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Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

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Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated, TEKS-aligned practice tests.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The main idea web helps students learn the vocabulary and organize the concepts related to integers. In each of the outer boxes, students should write one or more review words that describe the numbers.

Understand Vocabulary

Use the following explanations to help students learn the preview words.

> Many things are alike in some ways and different in others. For example, a lizard and a snake are both reptiles. They are alike because they are both cold-blooded. But a lizard has legs and a snake does not. In that way they are different. When we think about how things are alike and how they are different, we compare and contrast them.
>
> To help you compare and contrast ideas in this chapter, you will use a **Venn diagram**.

Active Reading

Integrating the ELPS

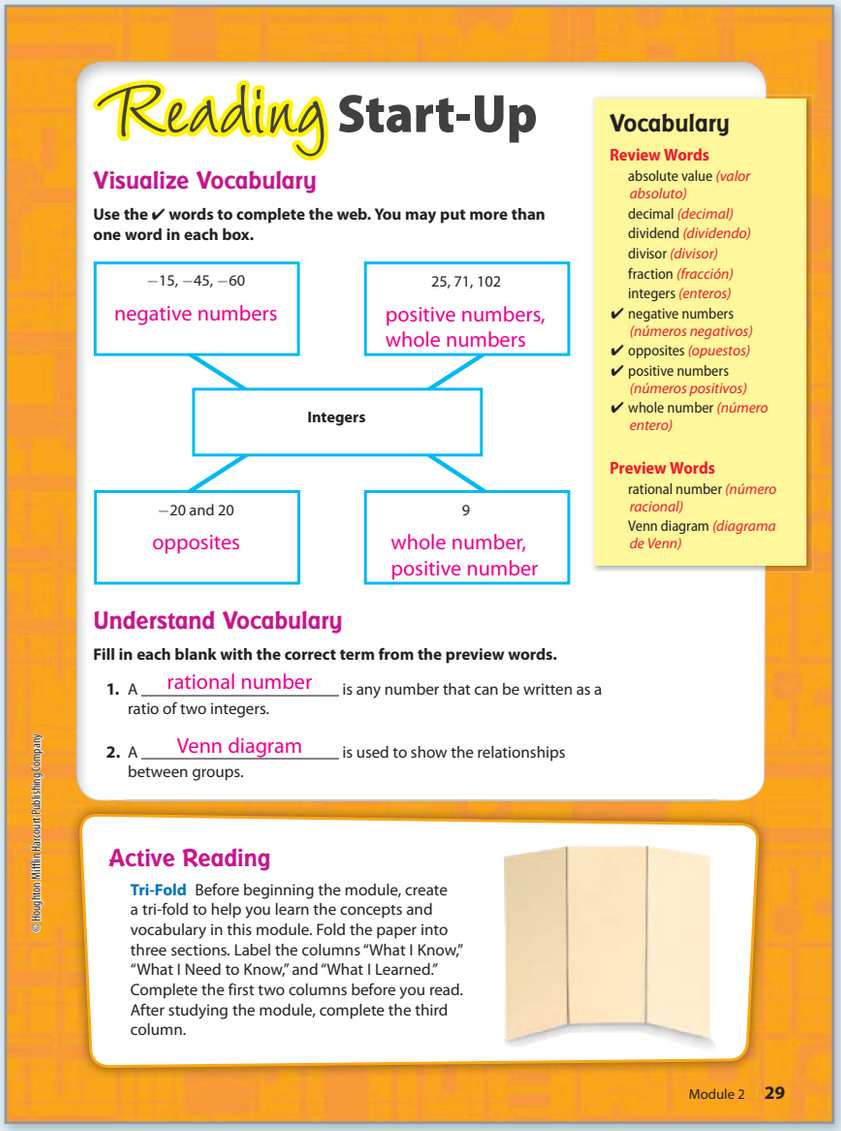
Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

ELPS **c.4.D** Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary to enhance comprehension of written text.

Additional Resources

Differentiated Instruction

- Reading Strategies ELL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the web. You may put more than one word in each box.

−15, −45, −60 — negative numbers

25, 71, 102 — positive numbers, whole numbers

Integers

−20 and 20 — opposites

9 — whole number, positive number

Understand Vocabulary

Fill in each blank with the correct term from the preview words.

1. A rational number is any number that can be written as a ratio of two integers.
2. A Venn diagram is used to show the relationships between groups.

Active Reading

Tri-Fold Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label the columns "What I Know," "What I Need to Know," and "What I Learned." Complete the first two columns before you read. After studying the module, complete the third column.

Vocabulary

Review Words
- absolute value (valor absoluto)
- decimal (decimal)
- dividend (dividendo)
- divisor (divisor)
- fraction (fracción)
- integers (enteros)
- ✔ negative numbers (números negativos)
- ✔ opposites (opuestos)
- ✔ positive numbers (números positivos)
- ✔ whole number (número entero)

Preview Words
- rational number (número racional)
- Venn diagram (diagrama de Venn)

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Module 2 29

Grades 6–8 TEKS

Before	In this module	After
Students understand whole numbers and integers: • identify a number and its opposite • compare and order integers using a number line • find the absolute value of a number	Students classify, order, and perform computations with rational numbers: • classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers • identify opposites and absolute values of rational numbers • compare and order a set of rational numbers arising from mathematical and real-world contexts	Students will connect rational numbers and integers: • describe relationships between sets and subsets of rational numbers • perform operations with rational numbers • locate, compare, and order rational numbers using a number line

Unpacking the TEKS

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Texas Essential Knowledge and Skills

Content Focal Areas

TEKS Number and Operations—6.2

The student applies mathematical process standards to represent and use rational numbers in a variety of forms.

Integrating the ELPS

ELPS c.4.F Use visual and contextual support . . . to read grade-appropriate content area text . . . and develop vocabulary . . . to comprehend increasingly challenging language.

Go online to see a complete unpacking of the TEKS.

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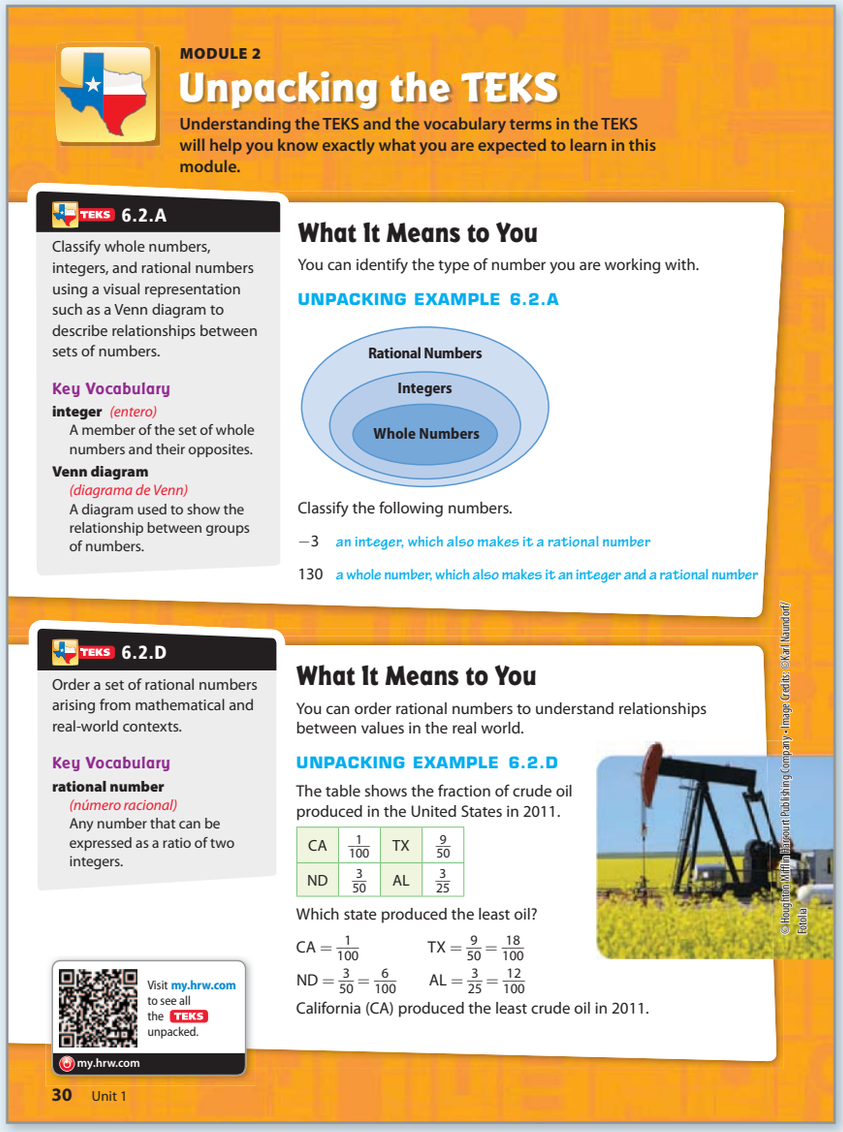

MODULE 2

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 6.2.A

Classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers.

Key Vocabulary

integer *(entero)*
A member of the set of whole numbers and their opposites.

Venn diagram *(diagrama de Venn)*
A diagram used to show the relationship between groups of numbers.

What It Means to You

You can identify the type of number you are working with.

UNPACKING EXAMPLE 6.2.A

Classify the following numbers.

−3 *an integer, which also makes it a rational number*

130 *a whole number, which also makes it an integer and a rational number*

TEKS 6.2.D

Order a set of rational numbers arising from mathematical and real-world contexts.

Key Vocabulary

rational number *(número racional)*
Any number that can be expressed as a ratio of two integers.

What It Means to You

You can order rational numbers to understand relationships between values in the real world.

UNPACKING EXAMPLE 6.2.D

The table shows the fraction of crude oil produced in the United States in 2011.

CA	$\frac{1}{100}$	TX	$\frac{9}{50}$
ND	$\frac{3}{50}$	AL	$\frac{3}{25}$

Which state produced the least oil?

$CA = \frac{1}{100}$ $TX = \frac{9}{50} = \frac{18}{100}$

$ND = \frac{3}{50} = \frac{6}{100}$ $AL = \frac{3}{25} = \frac{12}{100}$

California (CA) produced the least crude oil in 2011.

Visit my.hrw.com to see all the TEKS unpacked.

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Grade 6 TEKS	Lesson 2.1	Lesson 2.2	Lesson 2.3
TEKS 6.2.A Classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers.	★		
TEKS 6.2.B Identify a number, its opposite, and its absolute value.		★	
TEKS 6.2.D Order a set of rational numbers arising from mathematical and real-world contexts.			★
TEKS 6.2.E Extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$.	★		

LESSON

2.1 Classifying Rational Numbers

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.A

Classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers.

TEKS Number and operations—6.2.E

Extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$.

Mathematical Processes

TEKS 6.1.G

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

ADDITIONAL EXAMPLE 1

Write each rational number as $\frac{a}{b}$.

A) $1\frac{3}{8}$ $\frac{11}{8}$

B) 0.75 $\frac{75}{100}$

C) 12 $\frac{12}{1}$

D) -9 $\frac{-9}{1}$

Interactive Whiteboard

Interactive example available online

my.hrw.com

Engage

ESSENTIAL QUESTION

How can you classify rational numbers? You classify numbers according to their characteristics. Rational numbers can be written as a quotient of two integers, so rational numbers can be fractions, decimals, integers, or whole numbers.

Motivate the Lesson

Ask: Biologists classify animals based on shared characteristics. For example, the horned lizard is an animal, a reptile, a lizard, and a gecko. How can you classify the number -12?

Explore

EXPLORE ACTIVITY

Focus on Modeling

It may be helpful to give each student 3 paper squares and a pair of scissors to work through the Explore Activity with physical models.

Explain

EXAMPLE 1

Focus on Math Connections

Point out to students that part of the definition of a rational number is that the denominator, b, cannot equal zero. This is because division by zero is undefined. It is possible to divide 0 pizza between 3 people; each person would get 0 pizza. But 3 pizzas shared by 0 people is meaningless.

Questioning Strategies Mathematical Processes

- How do you decide which number to use for the denominator when you are rewriting a decimal as a fraction? You use the place value of the digit farthest to the right. For example, if the decimal has 2 places, use 100 for the denominator.
- What are three equivalent expressions for $3\frac{2}{5}$? You can write it as a mixed number, as an improper faction, and as a decimal. $3\frac{2}{5}$; $\frac{17}{5}$; 3.4

YOUR TURN

Avoid Common Errors

Remind students that when writing a negative integer as a fraction, they need to include the negative sign with the fraction.

Talk About It

Check for Understanding

Ask: How can you show that a number is a rational number? by writing it as a fraction where the numerator and the denominator are both integers, and the denominator is not equal to 0

LESSON 2.1

Classifying Rational Numbers

TEKS Number and operations—6.2.A Classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers. *Also 6.2.E.*

ESSENTIAL QUESTION

How can you classify rational numbers?

EXPLORE ACTIVITY Real World TEKS 6.2.E

Representing Division as a Fraction

Alicia and her friends Brittany, Kenji, and Ellis are taking a pottery class. The four friends have to share 3 blocks of clay. How much clay will each of them receive if they divide the 3 blocks evenly?

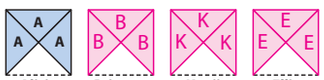
A The top faces of the 3 blocks of clay can be represented by squares. Use the model to show the part of each block that each friend will receive. Explain.

Divide each square into 4 equal pieces. Write each person's first initial on a piece of each square.

B Each piece of one square is equal to what fraction of a block of clay?

$\frac{1}{4}$

C Explain how to arrange the pieces to model the amount of clay each person gets. Sketch the model.



Arrange each person's pieces in a square.

D What fraction of a square does each person's pieces cover? Explain.

$\frac{3}{4}$; there is room for 4 pieces in each square, but each person only has 3 pieces, so 3 of 4 or $\frac{3}{4}$ of the square is covered.

E How much clay will each person receive?

$\frac{3}{4}$ block of clay

F **Multiple Representations** How does this situation represent division?

Dividing the 3 blocks of clay equally among 4 friends represents $3 \div 4$.

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EXPLORE ACTIVITY *(cont'd)*

Reflect

1. **Communicate Mathematical Ideas** $3 \div 4$ can be written $\frac{3}{4}$. How are the dividend and divisor of a division expression related to the parts of a fraction?

The divisor is the denominator of the fraction and the dividend is the numerator.

2. **Analyze Relationships** How could you represent the division as a fraction if 5 people shared 2 blocks? if 6 people shared 5 blocks?

$\frac{2}{5}$; $\frac{5}{6}$

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Rational Numbers

A **rational number** is any number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

EXAMPLE 1 TEKS 6.2.A

Write each rational number as $\frac{a}{b}$.

A $3\frac{2}{5}$ — Convert the mixed number to a fraction greater than 1. $3\frac{2}{5} = \frac{17}{5}$

B 0.6 — The decimal is six tenths. Write as a fraction. $0.6 = \frac{6}{10}$

C 34 — Write the whole number as a fraction with a denominator of 1. $34 = \frac{34}{1}$

D -7 — Write the integer as a fraction with a denominator of 1. $-7 = \frac{-7}{1}$

Math Talk Mathematical Processes

What division is represented by the fraction $\frac{34}{1}$?

$34 \div 1$

YOUR TURN

Write each rational number as $\frac{a}{b}$.

3. -15 $\frac{-15}{1}$

4. 0.31 $\frac{31}{100}$

5. $4\frac{5}{9}$ $\frac{41}{9}$

6. 62 $\frac{62}{1}$

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

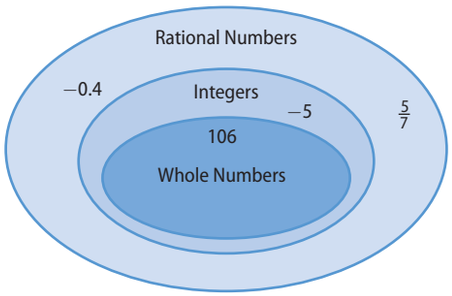
This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.G,** which calls for students to "justify mathematical ideas … using precise mathematical language." Students use the mathematical definition of a rational number to justify that numbers are rational numbers by rewriting them to meet the requirements of the definition.

Math Background

Rational numbers and Irrational numbers make up the set of Real Numbers. As this lesson states, rational numbers are numbers that can be expressed as a quotient of two integers. Irrational numbers are numbers that cannot be expressed as a quotient of two integers.

ADDITIONAL EXAMPLE 2

Use the Venn diagram to determine to which set or sets each number belongs. Place the numbers in the Venn diagram.



A) 106
whole numbers, integers, rational numbers

B) −5
integers, rational numbers

C) $\frac{5}{7}$
rational numbers

D) −0.4
rational numbers

Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Questioning Strategies

Mathematical Processes

- In a Venn diagram, explain what it means when a number is within a particular circle. It means that the number is part of the group represented by the circle. For example, a number within the Integers circle is an integer.
- The Venn diagram shows the Whole Numbers circle within the Integers circle. What does that tell you about whole numbers? about integers? Name an integer that is not a whole number. It means that all whole numbers are integers. Not all integers are whole numbers. −2 is an integer but not a whole number.

Focus on Math Connections

Mathematical Processes

Point out to students the connections among the sets shown in the Venn diagram. The Whole Numbers set is the smallest and most interior set, so it is part of both of the larger sets, Integers and Rational Numbers.

YOUR TURN

Talk About It

Check for Understanding

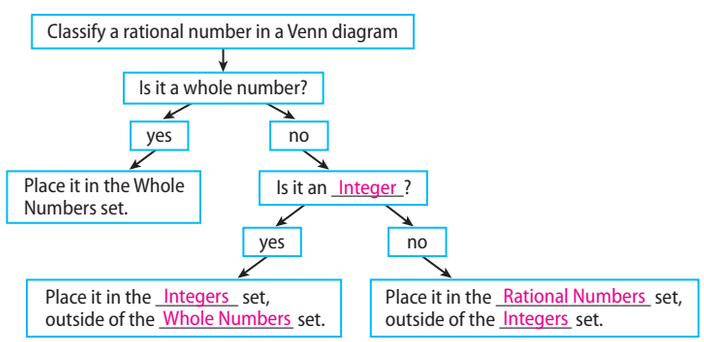
Ask: Suppose a student says that −15 is an integer, but it is not a rational number because it is not a fraction. Is the student correct? Explain. The student is not correct. Rational numbers can be integers because you can write -15 as $\frac{-15}{1}$.

Elaborate

Talk About It

Summarize the Lesson

Present the graphic organizer showing how to classify a rational number in a Venn diagram. Discuss each box in the graphic organizer and complete it with students.



GUIDED PRACTICE

Engage with the Whiteboard

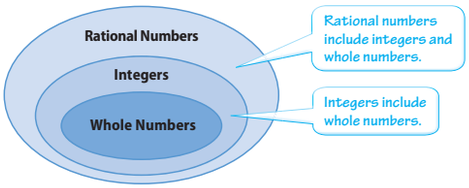
Have students suggest numbers that can be placed within the Venn diagram given for Exercises 5 and 6. Discuss what each placement tells them about the number.

Avoid Common Errors

Exercise 1 Remind students that Sarah counts as one of the classmates sharing the ribbon, so there are 5 students sharing.

Classifying Rational Numbers

A **Venn diagram** is a visual representation used to show the relationships between groups. The Venn diagram below shows how rational numbers, integers, and whole numbers are related.



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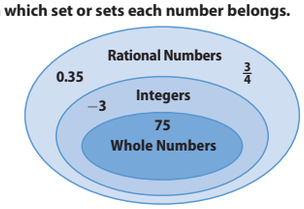
EXAMPLE 2

TEKS 6.2.A

Place each number in the Venn diagram. Then classify each number by indicating in which set or sets each number belongs.



A 75 — The number 75 belongs in the sets of whole numbers, integers, and rational numbers.

B −3 — The number −3 belongs in the sets of integers and rational numbers.

C $\frac{3}{4}$ — The number $\frac{3}{4}$ belongs in the set of rational numbers.

D 0.35 — The number 0.35 belongs in the set of rational numbers.

Reflect

7. **Analyze Relationships** Name two integers that are not also whole numbers.

Sample answers: −2, −3

8. **Analyze Relationships** Describe how the Venn diagram models the relationship between rational numbers, integers, and whole numbers.

All whole numbers are integers and rational numbers.

All integers are rational numbers.

My Notes

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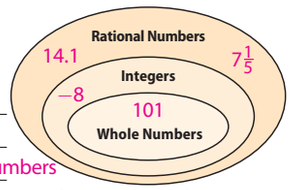
YOUR TURN

Place each number in the Venn diagram. Then classify each number by indicating in which set or sets it belongs.



9. 14.1 rational numbers

10. $7\frac{1}{5}$ rational numbers

11. −8 integers and rational numbers

12. 101 whole numbers, integers, and rational numbers

Guided Practice

1. Sarah and four friends are decorating picture frames with ribbon. They have 4 rolls of ribbon to share evenly. (Explore Activity 1)

a. How does this situation represent division?

4 rolls of ribbon divided evenly among the 5 friends. 4 ÷ 5.

b. How much ribbon does each person receive? $\frac{4}{5}$ roll

Write each rational number in the form $\frac{a}{b}$, where a and b are integers. (Example 1)

2. 0.7 $\frac{7}{10}$

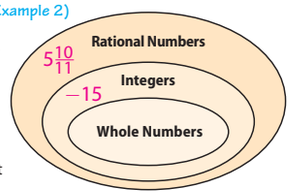
3. −29 $\frac{-29}{1}$

4. $8\frac{1}{3}$ $\frac{25}{3}$

Place each number in the Venn diagram. Then classify each number by indicating in which set or sets each number belongs. (Example 2)

5. −15 integers, rational numbers

6. $5\frac{10}{11}$ rational numbers



ESSENTIAL QUESTION CHECK-IN

7. How is a rational number that is not an integer different from a rational number that is an integer?

When written in the form $\frac{a}{b}$, non integer rational numbers have a denominator that does not divide evenly into the numerator.

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DIFFERENTIATE INSTRUCTION

Number Sense

Guide students to create a list of names for the different types of numbers. The list should include fractions, decimals, mixed numbers, positive fractions, negative fractions, whole numbers, counting numbers, and improper fractions. Have students give examples of each type that is mentioned. Then relate all the terms to a Rational Number Venn diagram.

Kinesthetic Experience

Give each student an index card. Have them write a rational number on the card. Encourage a variety of rational numbers: whole numbers, integers, fractions, and decimals. Then work with the class to create a Rational Number Venn diagram on which to place the cards. Use yarn to define an area for Rational Numbers, within that an area for Integers, and within the integer area, an area for Whole Numbers.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online homework assignment available

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2.1 LESSON QUIZ

TEKS **6.2.A, 6.2.E**

1. There are 2 pounds of peanuts to be divided evenly into 10 bags.

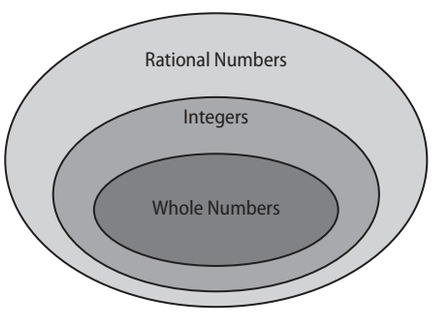
a. How does this situation represent division?

b. What fraction of a pound of peanuts will each bag get?

Write each rational number as $\frac{a}{b}$.

2. -27 **3.** 16 **4.** 0.15 **5.** $7\frac{1}{2}$

Use the Venn diagram to determine in which set or sets each number belongs. Place the numbers in the Venn diagram.



6. 18 **7.** -17 **8.** -2.3 **9.** $\frac{3}{8}$

Lesson Quiz available online

my.hrw.com

Answers

1. a. 2 pounds must be evenly divided among 10 bags. This represents the division $2 \div 10$.

b. $\frac{2}{10}$ or $\frac{1}{5}$ pound

2. $\frac{-27}{1}$ **4.** $\frac{15}{100}$

3. $\frac{16}{1}$ **5.** $\frac{15}{2}$

6. Whole Numbers, Integers, Rational Numbers

7. Integers, Rational Numbers

8. Rational Numbers

9. Rational Numbers

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS **6.2.A, 6.2.E**

Concepts & Skills	Practice
Explore Activity Representing Division as a Fraction	Exercises 1, 10, 14–16
Example 1 Rational Numbers	Exercises 2–4, 11
Example 2 Classifying Rational Numbers	Exercises 5, 6, 8, 9, 13, 14–16

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
8	**1** Recall of Information	**1.E** Create and use representations
9	**1** Recall of Information	**1.E** Create and use representations
10–12	**2** Skills/Concepts	**1.A** Everyday life
13	**1** Recall of Information	
14–16	**2** Skills/Concepts	**1.A** Everyday life
17	**3** Strategic Thinking H.O.T.	**1.A** Everyday life
18	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify
19	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
20	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

2.1 Independent Practice

TEKS 6.2.A, 6.2.E

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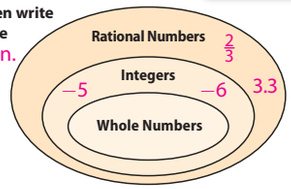
List two numbers that fit each description. Then write the numbers in the appropriate location on the Venn diagram. Sample answers are given.



8. Integers that are not whole numbers
−5, −6

9. Rational numbers that are not integers
$\frac{2}{3}$, 3.3

10. **Multistep** A nature club is having its weekly hike. The table shows how many pieces of fruit and bottles of water each member of the club brought to share.

Member	Pieces of Fruit	Bottles of Water
Baxter	3	5
Hendrick	2	2
Mary	4	3
Kendra	5	7

a. If the hikers want to share the fruit evenly, how many pieces should each person receive?
$\frac{14}{4}$, or $3\frac{1}{2}$ pieces of fruit

b. Which hikers received more fruit than they brought on the hike?
Baxter and Hendrick

c. The hikers want to share their water evenly so that each member has the same amount. How much water does each hiker receive?
$\frac{17}{4}$, or $4\frac{1}{4}$ bottles of water

11. Sherman has 3 cats and 2 dogs. He wants to buy a toy for each of his pets. Sherman has $22 to spend on pet toys. How much can he spend on each pet? Write your answer as a fraction and as an amount in dollars and cents.
$\frac{22}{5}$, or $4.40

12. A group of 5 friends are sharing 2 pounds of trail mix. Write a division problem and a fraction to represent this situation.
$2 \div 5; \frac{2}{5}$

13. **Vocabulary** A Venn diagram can represent set relationships visually.

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Financial Literacy **For 14–16, use the table. The table shows the Jason's utility bills for one month. Write a fraction to represent the division in each situation. Then classify each result by indicating the set or sets to which it belongs.**

March Bills	
Water	$35
Gas	$14
Electric	$108

14. Jason and his 3 roommates share the cost of the electric bill evenly.
$\$\frac{108}{4}$; whole numbers, integers, rational numbers

15. Jason plans to pay the water bill with 2 equal payments.
$\$\frac{35}{2}$; rational numbers

16. Jason owes $15 for last month's gas bill also. The total amount of the two gas bills is split evenly among the 4 roommates.
$\$\frac{29}{4}$; rational numbers

17. Lynn has a watering can that holds 16 cups of water, and she fills it half full. Then she waters her 15 plants so that each plant gets the same amount of water. How many cups of water will each plant get?
$\frac{8}{15}$ cup

H.O.T. FOCUS ON HIGHER ORDER THINKING

18. **Critique Reasoning** DaMarcus says the number $\frac{24}{6}$ belongs only to the set of rational numbers. Explain his error.
$24 \div 6 = 4$, which is a whole number. $\frac{24}{6}$ belongs to the set of whole numbers and the set of integers, as well as the set of rational numbers.

19. **Analyze Relationships** Explain how the Venn diagrams in this lesson show that all integers and all whole numbers are rational numbers.
The oval representing the set of integers and the oval representing the set of whole numbers are inside of the oval representing the set of rational numbers.

20. **Critical Thinking** Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain.
No; Every whole number is an integer.

Work Area

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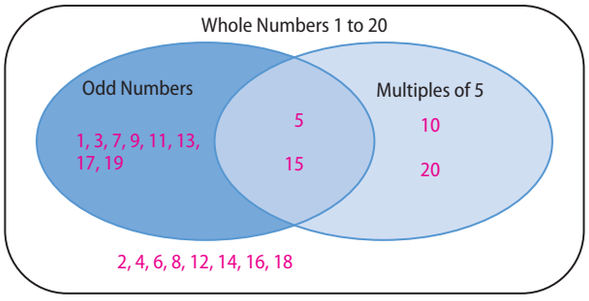
EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Use this activity to extend student understanding of sets, subsets, and Venn diagrams. Have students complete the Venn diagram by placing the numbers from 1 to 20 in the correct part of the diagram.



LESSON

2.2 Identifying Opposites and Absolute Value of Rational Numbers

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.B

Identify a number, its opposite, and its absolute value.

Mathematical Processes

TEKS 6.1.D

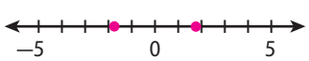
Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ADDITIONAL EXAMPLE 1

Alberto's average running time for 100 meters is $17\frac{1}{2}$ sec. Each day after he warms up, Alberto records his run time so he can compare it to his average time.

Day	Monday	Tuesday
Change in time	$2\frac{1}{4}$	$-1\frac{3}{4}$

Graph the change in time for Tuesday and its opposite.



Interactive Whiteboard
Interactive example available online

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Engage

ESSENTIAL QUESTION

How can you identify opposites and absolute values of rational numbers? Opposites are the same distance from 0 on the number line but on different sides of 0. Absolute value is the number's distance from 0.

Motivate the Lesson

Ask: Have you ever seen a picture of Death Valley? Death Valley is 282 feet below sea level. What is the opposite of 282 feet below sea level? How can you use positive and negative numbers to express values and their opposites? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Connect to Daily Life

Ask students if they have seen the tide come in, in person or in pictures or video. Point out to students that as the tides change so does sea level. At low tide, land that was below sea level may now be above sea level.

Explain

EXAMPLE 1

Focus on Math Connections

Emphasize that just as with positive and negative integers, the opposite of any negative rational number is a positive rational number, and the opposite of a positive rational number is a negative rational number.

Questioning Strategies Mathematical Processes

- Using the information presented in the table, can you find the price of the stock at the end of Wednesday? No, because the table does not give the starting price on Tuesday.
- How can you tell whether the stock gained value or lost value? Look at the sign of the number on the table. A positive number represents a gain; a negative number represents a loss.

YOUR TURN

Engage with the Whiteboard

Have a student plot the value for Tuesday, $1\frac{5}{8}$, and its opposite on the vertical number line given in Step 1.

Talk About It

Check for Understanding

Ask: How do you find the opposite of a negative number on a number line? Find the number that is the same distance from 0 on the right hand side of 0 on the number line.

LESSON 2.2

Identifying Opposites and Absolute Value of Rational Numbers

TEKS Number and operations—6.2.B Identify a number, its opposite, and its absolute value.

ESSENTIAL QUESTION How do you identify opposites and absolute value of rational numbers?

EXPLORE ACTIVITY Real World TEKS 6.2.B

Positive and Negative Rational Numbers

Recall that positive numbers are greater than 0. They are located to the right of 0 on a number line. Negative numbers are less than 0. They are located to the left of 0 on a number line.

Water levels with respect to sea level, which has elevation 0, may be measured at beach tidal basins. Water levels below sea level are represented by negative numbers.

A The table shows the water level at a tidal basin at different times during a day. Graph the level for each time on the number line.

Time	4 A.M. *A*	8 A.M. *B*	Noon *C*	4 P.M. *D*	8 P.M. *E*
Level (ft)	3.5	2.5	−0.5	−2.5	0.5

B How did you know where to graph −0.5? It is halfway between −1 and 0.

C At what time or times is the height closest to sea level? How do you know?
noon and 8 P.M.; they are each 0.5 units from 0.

D Which point is located halfway between −3 and −2? D

E Which point is the same distance from 0 as *D*? B

Reflect

1. **Communicate Mathematical Ideas** How would you graph −2.25? Would it be left or right of point *D*?
Graph the point halfway between −2.5 and −2; right

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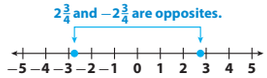
Rational Numbers and Opposites on a Number Line

You can find the opposites of rational numbers the same way you found the opposites of integers. Two rational numbers are opposites if they are the same distance from 0 but on different sides of 0.



EXAMPLE 1 Real World TEKS 6.2.B

Until June 24, 1997, the New York Stock Exchange priced the value of a share of stock in eighths, such as $27 1/8 or at $41 3/4. The change in value of a share of stock from day to day was also represented in eighths as a positive or negative number.

The table shows the change in value of a stock over two days. Graph the change in stock value for Wednesday and its opposite on a number line.

Day	Tuesday	Wednesday
Change in value ($)	1 5/8	−4 1/4

STEP 1 Graph the change in stock value for Wednesday on the number line.

The change in value for Wednesday is $-4\frac{1}{4}$.

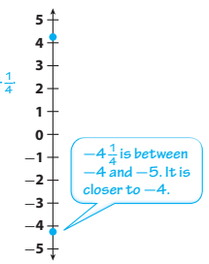
Graph a point $4\frac{1}{4}$ units below 0.

STEP 2 Graph the opposite of $-4\frac{1}{4}$.

The opposite of $-4\frac{1}{4}$ is the same distance from 0 but on the other side of 0.

The opposite of $-4\frac{1}{4}$ is $4\frac{1}{4}$.

The opposite of the change in stock value for Wednesday is $4\frac{1}{4}$.



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YOUR TURN

2. What are the opposites of 7, −3.5, 2.25, and $9\frac{1}{3}$?
-7, 3.5, -2.25, and $-9\frac{1}{3}$.

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.D,** which calls for students to "communicate mathematical ideas … using multiple representations … as appropriate." In the Explore Activity and Example 1, students use number lines to model the relationship between positive and negative numbers and absolute value. Using a number line, students can see that absolute values of opposites are equal because the opposites are the same distance from 0.

Math Background

Just as with integers, you can interpret absolute value of rational numbers as the magnitude of the number without regard to its sign. It measures the amount of change rather than the direction of change.

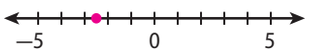
Many real-world situations involve absolute value. For example, the manufacturer of a 32-ounce box of cereal may have a 0.75 ounce tolerance in the weight of the contents of the box. This is an absolute value and means that the actual weight of the contents is acceptable between 32 − 0.75 and 32 + 0.75 ounces.

ADDITIONAL EXAMPLE 2

Eric tries to begin each day with $5.00 in his backpack. The table shows how much more or less than $5.00 he had in his backpack at the end of each day during a 3-day period.

Day	Mon	Tues	Wed
Amount ($)	1.25	−2.50	−0.75

Graph the amount more or less than $5 he had at the end of Tuesday.



$|-2.50| =$ 2.50

Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Questioning Strategies Mathematical Processes

- Is the point −5.4 above or below −3.2 on a vertical number line? Explain? It is below −3.2, because −5.4 is farther from 0 than −3.2.
- Which number in the table has the least absolute value? Explain. −0.8 because it is the closest number to 0.

Focus on Math Connections Mathematical Processes

Make sure students understand that distance from zero is always a nonnegative value because distance does not indicate a direction from zero.

YOUR TURN

Talk About It

Check for Understanding

Ask: How do you use a number line to find the absolute value of a number? Graph the number, and find the distance between the number and 0.

Connect to Daily Life Mathematical Processes

Discuss with students that financial debt is often discussed without any reference to the negative sign. A start-up tech company borrows $50,000 to develop a product. Then the company borrows another $25,000 to complete the project. Point out that the actual debt value is −$75,000 because it is money owed. When you talk about the amount of debt, the negative symbol is dropped because the word *debt* indicates that it is a negative value, an amount owed.

Elaborate

Talk About It

Summarize the Lesson

Ask: How is absolute value related to the concept of opposites? Opposites are on opposite sides of 0 on the number line, but they have the same absolute value.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–4, have students plot each number and its opposite on the number lines.

Avoid Common Errors

Exercises 1–4 If students have trouble estimating the position of a rational number on the number line, encourage them to first identify the two consecutive integers between which it lies.

Exercises 5–18 Caution students to read the directions carefully. For Exercises 5–10, they need to find the opposite, and for Exercises 12–17 they need to find the absolute value.

Absolute Values of Rational Numbers

You can also find the absolute value of a rational number the same way you found the absolute value of an integer. The absolute value of a rational number is the number's distance from 0 on the number line.

EXAMPLE 2

Real World

TEKS 6.2.B

The table shows the average low temperatures in January in one location during a five-year span. Find the absolute value of the average January low temperature in 2009.

Year	2008	2009	2010	2011	2012
Temperature (°C)	−3.2	−5.4	−0.8	3.8	−2

STEP 1 Graph the 2009 average January low temperature.

The 2009 average January low is −5.4 °C. Graph a point 5.4 units below 0.

STEP 2 Find the absolute value of −5.4.

−5.4 is 5.4 units from 0.

$|-5.4| = 5.4$

Reflect

3. **Communicate Mathematical Ideas** What is the absolute value of the average January low temperature in 2011? How do you know?

3.8; it is nonnegative, so the absolute value is the number itself.

Math On the Spot
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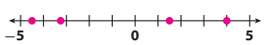
My Notes

−5.4 is between −5 and −6. Since 0.4 is less than $\frac{1}{2}$, it is a little closer to −5.

Math Talk
Mathematical Processes
How do you know where to graph −5.4?

YOUR TURN

Graph each number on the number line. Then use your number line to find each absolute value.



4. $-4.5; |-4.5| =$ 4.5

5. $1\frac{1}{2}; \left|1\frac{1}{2}\right| =$ $1\frac{1}{2}$

6. $4; |4| =$ 4

7. $-3\frac{1}{4}; \left|-3\frac{1}{4}\right| =$ $3\frac{1}{4}$

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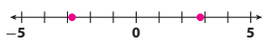
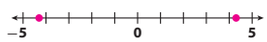
Guided Practice

Graph each number and its opposite on a number line. (Explore Activity and Example 1)

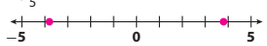
1. −2.8



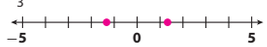
2. 4.3



3. $-3\frac{4}{5}$



4. $1\frac{1}{3}$



Find the opposite of each number. (Example 1)

5. 3.78 −3.78

6. $-7\frac{5}{12}$ $7\frac{5}{12}$

7. 0 0

8. 4.2 −4.2

9. 12.1 −12.1

10. 2.6 −2.6

11. **Vocabulary** Explain why 2.15 and −2.15 are opposites. (Example 1)

They are the same distance from 0 on the number line.

Find the absolute value of each number. (Example 2)

12. 5.23 5.23

13. $-4\frac{2}{11}$ $4\frac{2}{11}$

14. 0 0

15. $-6\frac{3}{5}$ $6\frac{3}{5}$

16. −2.12 2.12

17. 8.2 8.2

ESSENTIAL QUESTION CHECK-IN

18. How do you identify the opposite and the absolute value of a rational number?

Opposites are the same distance from zero on the number line in the opposite direction. Absolute value is the number's distance from zero.

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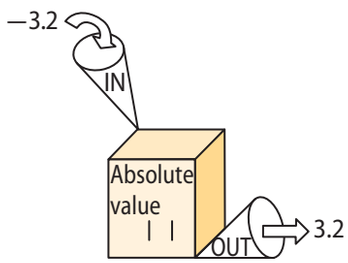
DIFFERENTIATE INSTRUCTION

Cognitive Strategies

To help students understand the concept of opposite integers, discuss what *opposite* means in contexts other than math. Have students generate a list of opposite terms (e.g., black and white, up and down, tall and short). Then have students name some pairs of opposite integers (e.g., 1 and −1, −13 and 13).

Visual Cues

Draw the absolute value symbols on the board and create a function machine around them. Have students take turns putting in a number and showing the number that comes out. Suggest students think of the absolute value symbols as squeezing the negative sign from negative numbers.



Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

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Online Assessment and Intervention

Online homework assignment available

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2.2 LESSON QUIZ

TEKS 6.2.B

1. The table shows how the rainfall varied each of three months compared with the average rainfall for that month.

Month	Jun	Jul	Aug
Rainfall variation (in.)	−1.2	3.25	−0.5

a. Graph each month's rainfall variation and its opposite on a number line.

b. For which month did the variation differ the most from the monthly average? Explain.

Write the absolute value of each number.

2. 9.01 **3.** $-\frac{2}{8}$ **4.** −8.7 **5.** $3\frac{3}{4}$

Lesson Quiz available online

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Answers

1. a.

b. July; because July had 3.25 more inches than the average June had about 1 inch less than average, and August had 0.5 inches less than average.

2. 9.01

3. $\frac{2}{8}$

4. 8.7

5. $3\frac{3}{4}$

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS 6.2.B

Concepts & Skills	Practice
Explore Activity Positive and Negative Rational Numbers	Exercises 1–4
Example 1 Rational Numbers and Opposites on a Number Line	Exercises 1–11, 19, 20
Example 2 Absolute Values of Rational Numbers	Exercises 12–17, 21

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
19	**2** Skills/Concepts	**1.A** Everyday life
20	**2** Skills/Concepts	**1.F** Analyze relationships
21	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments
22	**2** Skills/Concepts	**1.F** Analyze relationships
23	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments
24	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
25	**3** Strategic Thinking H.O.T.	**1.D** Multiple representations
26	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

2.2 Independent Practice

TEKS 6.2.B

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19. **Financial Literacy** A store's balance sheet represents the amounts customers owe as negative numbers and credits to customers as positive numbers.

Customer	Girardi	Lewis	Stein	Yuan	Wenner
Balance ($)	−85.23	20.44	−116.33	13.50	−9.85

a. Write the opposite of each customer's balance.

Girardi $85.23, Lewis −$20.44, Stein $116.33, Yuan −$13.50, Wenner $9.85

b. Mr. Yuan wants to use his credit to pay off the full amount that another customer owes. Which customer's balance does Mr. Yuan have enough money to pay off? Wenner

c. Which customer's balance would be farthest from 0 on a number line? Explain.

Stein; when you find the absolute value of each balance, Stein's is the greatest.

20. **Multistep** Trina and Jessie went on a vacation to Hawaii. Trina went scuba diving and reached an elevation of −85.6 meters, which is below sea level. Jessie went hang-gliding and reached an altitude of 87.9 meters, which is above sea level.

a. Who is closer to the surface of the ocean? Explain.

Trina; $|-85.6|$ is less than $|87.9|$

b. Trina wants to hang-glide at the same number of meters above sea level as she scuba-dived below sea level. Will she fly higher than Jessie did? Explain.

No; the opposite of −85.6 meters is 85.6 meters, which is less than 87.9 meters.

21. **Critical Thinking** Carlos finds the absolute value of −5.3, and then finds the opposite of his answer. Jason finds the opposite of −5.3, and then finds the absolute value of his answer. Whose final value is greater? Explain.

Jason's; Carlos finds $|-5.3| = 5.3$. Then he finds the opposite, which is −5.3. Jason finds the opposite of −5.3, which is 5.3. Then he finds $|5.3|$, which is 5.3.

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22. **Explain the Error** Two students are playing a math game. The object of the game is to make the least possible number by arranging the given digits on a card. In the first round, each player will use the digits 3, 5, and 7 to fill in the card.

$|-75.3|$

a. One student arranges the numbers on the card as shown. What was this students mistake?

The student made the least negative number that could be formed with the given digits but did not take into account the absolute value symbols.

b. What is the least possible number? $|-35.7|$

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

23. **Analyze Relationships** If you plot the point −8.85 on a number line, would you place it to the left or right of −8.8? Explain.

to the left; 8.85 is greater than 8.8, so −8.85 is farther from 0 on the number line.

24. **Make a Conjecture** If the absolute value of a negative number is 2.78, what is the distance on the number line between the number and its absolute value? Explain your answer.

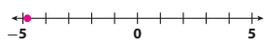
5.56; both values are 2.78 units from 0, and in opposite directions. So the distance is $2(2.78) = 5.56$.

25. **Multiple Representations** The deepest point in the Indian Ocean is the Java Trench, which is 25,344 feet below sea level. Elevations below sea level are represented by negative numbers.

a. Write the elevation of the Java Trench. −25,344 ft

b. A mile is 5,280 feet. Between which two integers is the elevation in miles? −5 and −4

c. Graph the elevation of the Java Trench in miles.



26. **Draw Conclusions** A number and its absolute value are equal. If you subtract 2 from the number, the new number and its absolute value are not equal. What do you know about the number? What is a possible number that satisfies these conditions?

It is greater than or equal to 0 and less than 2; 1 is a possible solution.

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EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity On Monday morning the opening price of a stock was $10. Complete the table to find the closing price of the stock on Friday afternoon. What was the closing price?

The closing price on Friday was $11.75.

Day	Change	Closing price calculation	Closing price
Monday	$2\frac{1}{8}$	$10 + 2\frac{1}{8}$	$12\frac{1}{8}$
Tuesday	$-1\frac{2}{8}$	$12\frac{1}{8} - 1\frac{2}{8}$	$10\frac{7}{8}$
Wednesday	$-\frac{2}{8}$	$10\frac{7}{8} - \frac{2}{8}$	$10\frac{5}{8}$
Thursday	$\frac{1}{8}$	$10\frac{5}{8} + \frac{1}{8}$	$10\frac{6}{8}$
Friday	1	$10\frac{6}{8} + 1$	$11\frac{6}{8}$

LESSON

2.3 Comparing and Ordering Rational Numbers

Texas Essential Knowledge and Skills

The student is expected to:

TEKS Number and operations—6.2.D

Order a set of rational numbers arising from mathematical and real-world contexts.

Mathematical Processes

TEKS 6.1.D

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ADDITIONAL EXAMPLE 1

A) Order 0.3, $\frac{2}{5}$, 0.85, 0.09, $\frac{3}{4}$, and $\frac{3}{20}$ from least to greatest.

$0.09, \frac{3}{20}, 0.3, \frac{2}{5}, \frac{3}{4}, 0.85$

B) Order 0.4, $\frac{1}{3}$, and $\frac{5}{6}$ from least to greatest.

$\frac{1}{3}, 0.4, \frac{5}{6}$

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Interactive example available online

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Animated Math
Ordering Rational Numbers

Students build fluency with ordering rational numbers in an engaging scoring game with a dynamic number line.

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Engage

ESSENTIAL QUESTION

How do you compare and order rational numbers? You can write them as equivalent decimals and then compare them.

Motivate the Lesson

Ask: Suppose you like to drink iced tea on a hot day. You can choose between two glasses, one with $\frac{1}{2}$ cup of tea or one with 0.6 cup of tea. Which glass contains more tea? Begin the Explore Activity to find out how to compare these rational numbers.

Explore

EXPLORE ACTIVITY

Focus on Modeling Mathematical Processes

Point out to students that the scale used for the number line is tenths. Every other fraction on the number line has a denominator of 5 because the labels are in simplest form. For example, $\frac{2}{10}$ simplifies to $\frac{1}{5}$.

Integrating the ELPS ELPS c.2.I.4 ELL

You may want to pair English learners with a partner for Explore Activity 1 to help them develop their language skills.

Explain

EXAMPLE 1

Questioning Strategies Mathematical Processes

- How can you compare two fractions? Compare their equivalent decimals or rewrite them with common denominators and compare the numerators.
- How can you compare a fraction with a decimal? Rewrite them so both are decimals or both are fractions.
- In B Step 2, if you use a different common denominator other than 60, will the order be the same? Justify your answer. Yes, as long as the fractions are equivalent to the original value, the order will remain the same.

Engage with the Whiteboard

Have students plot and label the values in A on the given number line with both fractional and decimal equivalents.

YOUR TURN

Connect Multiple Representations Mathematical Processes

Point out that writing rational numbers as equivalent decimals or fractions does not change the value of the rational number. It just makes it easier to compare rational numbers.

LESSON 2.3

Comparing and Ordering Rational Numbers

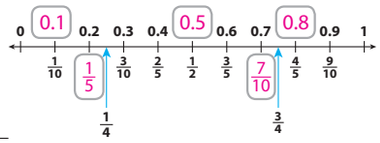
TEKS **Number and operations—6.2.D** Order a set of rational numbers arising from mathematical and real-world contexts.

ESSENTIAL QUESTION How do you compare and order rational numbers?

EXPLORE ACTIVITY TEKS 6.2.D

Equivalent Fractions and Decimals

Fractions and decimals that represent the same value are *equivalent*. The number line shows equivalent fractions and decimals from 0 to 1.



A Complete the number line by writing the missing decimals or fractions.

B Use the number line to find a fraction that is equivalent to 0.25. Explain.

$\frac{1}{4}$; 0.25 and $\frac{1}{4}$ both represent the point halfway between 0.2 and 0.3.

C Explain how to use a number line to find a decimal equivalent to $1\frac{7}{10}$.

$1\frac{7}{10}$ is a mixed number equal to $1 + \frac{7}{10}$. $\frac{7}{10}$ is equivalent to 0.7. $1 + 0.7$ is equal to 1.7, so $1\frac{7}{10}$ is equivalent to 1.7.

D Use the number line to complete each statement.

$0.2 = \frac{1}{5}$ $\quad 0.3 = \frac{3}{10}$ $\quad 0.75 = \frac{3}{4}$ $\quad 1.25 = 1\frac{1}{4}$

Reflect

1. **Communicate Mathematical Ideas** How does a number line represent equivalent fractions and decimals?

A decimal and fraction that represent the same point on the number line are equivalent.

2. Name a decimal between 0.4 and 0.5.

Sample answers: 0.42, 0.47

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Ordering Fractions and Decimals

You can order fractions and decimals by rewriting the fractions as equivalent decimals or by rewriting the decimals as equivalent fractions.

EXAMPLE 1 TEKS 6.2.D

A Order 0.2, $\frac{3}{4}$, 0.8, $\frac{1}{2}$, $\frac{1}{4}$, and 0.4 from least to greatest.

STEP 1 Write the fractions as equivalent decimals.

$\frac{1}{4} = 0.25$ $\quad \frac{1}{2} = 0.5$ $\quad \frac{3}{4} = 0.75$

STEP 2 Use the number line to write the decimals in order.

$0.2 < 0.25 < 0.4 < 0.5 < 0.75 < 0.8$

The numbers from least to greatest are 0.2, $\frac{1}{4}$, 0.4, $\frac{1}{2}$, $\frac{3}{4}$, 0.8.

B Order $\frac{1}{12}$, $\frac{2}{3}$, and 0.35 from least to greatest.

STEP 1 Write the decimal as an equivalent fraction.

$0.35 = \frac{35}{100} = \frac{7}{20}$

60 is a multiple of the denominators of all three fractions.

STEP 2 Find equivalent fractions with 60 as the common denominator.

$\frac{1}{12} = \frac{5}{60}$ (×5) $\quad \frac{2}{3} = \frac{40}{60}$ (×20) $\quad \frac{7}{20} = \frac{21}{60}$ (×3)

STEP 3 Order fractions with common denominators by comparing the numerators.

$5 < 21 < 40$

The fractions in order from least to greatest are $\frac{5}{60}$, $\frac{21}{60}$, $\frac{40}{60}$.

The numbers in order from least to greatest are $\frac{1}{12}$, 0.35, and $\frac{2}{3}$.

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YOUR TURN

Order the fractions and decimals from least to greatest.

3. 0.85, $\frac{3}{5}$, 0.15, $\frac{7}{10}$ $\quad 0.15, \frac{3}{5}, \frac{7}{10}, 0.85$

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PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.D,** which calls for students to "communicate mathematical ideas … using multiple representations … as appropriate." In the Explore Activity and both Examples, students compare rational numbers by writing and graphing their equivalent fractions or decimals. Thus students are ordering rational numbers using multiple representations.

Math Background

The process for writing equivalent fractions is based on two properties.

(1) A nonzero number divided by itself is 1, or $\frac{a}{a} = 1, a \neq 0$.

(2) $1 \cdot x = x$.

The process of writing an equivalent fraction by dividing the numerator and denominator by the same number is also based on the property that $\frac{ac}{bd} = \frac{a}{b} \cdot \frac{c}{d}, (b, d \neq 0)$.

ADDITIONAL EXAMPLE 2

Mr. Williams bought four boards. Each board was supposed to be 6 feet long. The table shows the difference between the length of each board and 6 feet.

Board 1	Board 2	Board 3	Board 4
−0.5 in.	$1\frac{1}{4}$ in.	0.25 in.	$\frac{3}{8}$ in.

Order the numbers from least to greatest.

$-0.5, 0.25, \frac{3}{8}, 1\frac{1}{4}$

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Interactive example available online

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EXAMPLE 2

Avoid Common Errors

Make sure that students understand that the table does not give any information about actual running times. It only shows how each runner's time differs from the average running time. Also be sure students understand that the fastest time is the farthest below the average.

Questioning Strategies Mathematical Processes

- How can you determine whose time was closest to the average time? Since 0 represents no difference from the average, find the time closest to 0. John's time is closest to 0.
- Why is the average time shown to be 0 on the number line? The numbers on the number line represent differences from the average time. The point 0 on the number line represents 0 or no difference from the average.

Engage with the Whiteboard

Have students take turns writing the converted fractions on the table and plotting points on the graph. As they plot a point, have them write the runner's name below it. Then ask students to list the runners from fastest to slowest according to their time.

YOUR TURN

Focus on Reasoning Mathematical Processes

Ask students to look at the table and determine who biked fastest and who biked slowest without making any calculations. Ask students to explain their reasoning. Help students see that the fastest biker will have the least time in minutes and the slowest biker will have the greatest.

Elaborate

Talk About It

Summarize the Lesson

Ask: How can a number line help you order rational numbers? Once you have graphed the numbers, the numbers will be in order from least to greatest from left to right.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–9, have students take turns writing an equivalent fraction or decimal for each number, while showing their work and explaining their reasoning. Discuss other possible equivalent fractions or decimals that students may have written.

Focus on Communication Mathematical Processes

For Exercises 12–20, have students discuss the methods they used to order the numbers.

Avoid Common Errors

Exercise 9 Point out that $\frac{6}{8} = \frac{3}{4}$, and they probably know the decimal equivalent for $\frac{3}{4}$.

Exercises 15–18 Caution students not to drop the negative sign when converting negative rational numbers.

Ordering Rational Numbers

You can use a number line to order positive and negative rational numbers.

EXAMPLE 2

Real World

TEKS 6.2.D

Five friends completed a triathlon that included a 3-mile run, a 12-mile bike ride, and a $\frac{1}{2}$-mile swim. To compare their running times they created a table that shows the difference between each person's time and the average time, with negative numbers representing times less than the average.

Runner	John	Sue	Anna	Mike	Tom
Time above or below average (minutes)	$\frac{1}{2}$	1.4	$-1\frac{1}{4}$	−2.0	1.95

Order the numbers from greatest to least.

STEP 1 Write the fractions as equivalent decimals.

$\frac{1}{2} = 0.5 \quad -1\frac{1}{4} = -1.25$

STEP 2 Use the number line to write the decimals in order.

$1.95 > 1.4 > 0.5 > -1.25 > -2.0$

The numbers in order from greatest to least are $1.95, 1.4, \frac{1}{2}, -1\frac{1}{4}, -2.0$.

Reflect

4. **Communicate Mathematical Ideas** Describe a different way to order the numbers.

Convert the decimals to fractions. $1.4 = 1\frac{4}{10}$; $-2.0 = -\frac{2}{1}$; $1.95 = 1\frac{95}{100}$; find a common denominator and compare the whole numbers and then the numerators.

YOUR TURN

5. To compare their bike times, the friends created a table that shows the difference between each person's time and the average bike time. Order the bike times from least to greatest.

Biker	John	Sue	Anna	Mike	Tom
Time above or below average (minutes)	−1.8	1	$1\frac{2}{5}$	$1\frac{9}{10}$	−1.25

$-1.8, -1.25, 1, 1\frac{2}{5}, 1\frac{9}{10}$

Math On the Spot
my.hrw.com

Math Talk
Mathematical Processes
Who was the fastest runner? Explain.

Mike; he finished running in the least amount of time.

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

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Guided Practice

Find the equivalent fraction or decimal for each number. (Explore Activity 1)

1. 0.6 = $\frac{3}{5}$

2. $\frac{1}{4}$ = 0.25

3. 0.9 = $\frac{9}{10}$

4. 0.1 = $\frac{1}{10}$

5. $\frac{3}{10}$ = 0.3

6. 1.4 = $1\frac{2}{5}$

7. $\frac{4}{5}$ = 0.8

8. 0.4 = $\frac{2}{5}$

9. $\frac{6}{8}$ = 0.75

Use the number line to order the fractions and decimals from least to greatest. (Example 1)

10. 0.75, $\frac{1}{2}$, 0.4, and $\frac{1}{5}$

$\frac{1}{5}, 0.4, \frac{1}{2}, 0.75$

11. The table shows the lengths of fish caught by three friends at the lake last weekend. Write the lengths in order from greatest to least. (Example 1)

$12\frac{3}{4}, 12.7, 12\frac{3}{5}$

Lengths of Fish (cm)		
Emma	Anne	Emily
12.7	$12\frac{3}{5}$	$12\frac{3}{4}$

List the fractions and decimals in order from least to greatest. (Example 1, Example 2)

12. 2.3, $2\frac{4}{5}$, 2.6

$2.3, 2.6, 2\frac{4}{5}$

13. 0.5, $\frac{3}{16}$, 0.75, $\frac{5}{48}$

$\frac{5}{48}, \frac{3}{16}, 0.5, 0.75$

14. 0.5, $\frac{1}{5}$, 0.35, $\frac{12}{25}$, $\frac{4}{5}$

$\frac{1}{5}, 0.35, \frac{12}{25}, 0.5, \frac{4}{5}$

15. $\frac{3}{4}, -\frac{7}{10}, -\frac{3}{4}, \frac{8}{10}$

$-\frac{3}{4}, -\frac{7}{10}, \frac{3}{4}, \frac{8}{10}$

16. $-\frac{3}{8}, \frac{5}{16}, -0.65, \frac{2}{4}$

$-0.65, -\frac{3}{8}, \frac{5}{16}, \frac{2}{4}$

17. $-2.3, -2\frac{4}{5}, -2.6$

$-2\frac{4}{5}, -2.6, -2.3$

18. $-0.6, -\frac{5}{8}, -\frac{7}{12}, -0.72$

$-0.72, -\frac{5}{8}, -0.6, -\frac{7}{12}$

19. $1.45, 1\frac{1}{2}, 1\frac{1}{3}, 1.2$

$1.2, 1\frac{1}{3}, 1.45, 1\frac{1}{2}$

20. $-0.3, 0.5, 0.55, -0.35$

$-0.35, -0.3, 0.5, 0.55$

ESSENTIAL QUESTION CHECK-IN

21. Explain how to compare 0.7 and $\frac{5}{8}$.

Convert the fraction to a decimal. $\frac{5}{8} = 0.625$. Compare by using place value or graphing both numbers on a number line. $0.7 > \frac{5}{8}$

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DIFFERENTIATE INSTRUCTION

Cooperative Learning

Have students work in pairs to order each set from least to greatest. Instruct the pairs to order one set using decimals and one set using fractions. Invite pairs to explain how they chose which set to order with decimals and which to order with fractions.

Set 1: $0.3, -1.7, -1\frac{3}{5}, -1\frac{7}{20}, -\frac{1}{2}, 0.05$

Set 2: $-0.2, 0.5, 1\frac{1}{3}, -2\frac{1}{6}, \frac{4}{15}, 0.1$

Set 1: $-1.7, -1\frac{3}{5}, -\frac{1}{2}, 0.05, 0.3, 1\frac{7}{20}$

Set 2: $-2\frac{1}{6}, -0.2, 0.1, \frac{4}{15}, 0.5, 1\frac{1}{3}$

Multiple Representations

Have students sketch a number line for Exercises 12 and 17 in the Guided Practice. Have them determine what scale and the range of numbers to use on each number line. Then have them plot and label the points on the number line.

Exercise 12

Exercise 17

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners ELL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

2.3 LESSON QUIZ

TEKS **6.2.D**

Andy, Dana, and Becky each worked on homework for an hour. The table shows what part of homework time each spent on math and English.

	Math	English
Andy	$\frac{1}{2}$	0.2
Dana	0.35	$\frac{3}{5}$
Becky	$\frac{3}{8}$	0.4

1. Who spent the most time on math?
2. Order the times spent on English from least to greatest.
3. List all the fractions and decimals given in the table from least to greatest.
4. Order the set of fractions and decimals below from least to greatest.

$0.15, -\frac{4}{5}, -1.25, 1.03, 1\frac{5}{6}, -\frac{7}{20}$

Lesson Quiz available online

my.hrw.com

Answers

1. Andy
2. 0.2, 0.4, $\frac{3}{5}$
3. 0.2, 0.35, $\frac{3}{8}$, 0.4, $\frac{1}{2}$, $\frac{3}{5}$
4. $-1.25, -\frac{4}{5}, -\frac{7}{20}, 0.15, 1.03, 1\frac{5}{6}$

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS **6.2.D**

Concepts & Skills	Practice
Explore Activity Equivalent Fractions and Decimals	Exercises 1–9
Example 1 Ordering Fractions and Decimals	Exercises 10–20
Example 2 Ordering Rational Numbers	Exercises 10–20, 22

Exercise	Depth of Knowledge (D.O.K.)	TEKS Mathematical Processes
22	**2** Skills/Concepts	**1.A** Everyday life
23	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
24	**2** Skills/Concepts	**1.A** Everyday life
25	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
26	**3** Strategic Thinking H.O.T.	**1.F** Analyze relationships
27	**3** Strategic Thinking H.O.T.	**1.G** Explain and justify arguments

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

Name _____ Class _____ Date _____

2.3 Independent Practice

TEKS 6.2.D

Personal Math Trainer
Online Assessment and Intervention

22. Rosa and Albert receive the same amount of allowance each week. The table shows what part of their allowance they each spent on video games and pizza. Use a number line to help you compare.

	Video games	Pizza
Rosa	0.4	$\frac{2}{5}$
Albert	$\frac{1}{2}$	0.25

a. Who spent more of their allowance on video games? Write an inequality to compare the portion spent on video games.

Albert; $\frac{1}{2} > 0.4$ or $0.4 < \frac{1}{2}$

b. Who spent more of their allowance on pizza? Write an inequality to compare the portion spent on pizza.

Rosa; $\frac{2}{5} > 0.25$ or $0.25 < \frac{2}{5}$

c. **Draw Conclusions** Who spent the greater part of their total allowance? How do you know?

Rosa; she spent $0.4 + \frac{2}{5} = 0.4 + 0.4 = 0.8$;

Albert spent $0.25 + \frac{1}{2} = 0.25 + 0.5 = 0.75$

23. A group of friends is collecting aluminum for a recycling drive. Each person who donates at least 4.25 pounds of aluminum receives a free movie coupon. The weight of each person's donation is shown in the table.

	Brenda	Claire	Jim	Micah	Peter
Weight (lb)	4.3	5.5	$6\frac{1}{6}$	$\frac{15}{4}$	$4\frac{3}{8}$

a. Order the weights of the donations from greatest to least.

$6\frac{1}{6}$, 5.5, $4\frac{3}{8}$, 4.3, $\frac{15}{4}$

b. Which of the friends will receive a free movie coupon? Which will not?

Claire, Peter, Brenda, and Jim; Micah

c. **What If?** Would the person with the smallest donation win a movie coupon if he or she had collected $\frac{1}{2}$ pound more of aluminum? Explain.

Yes; the smallest donation is $\frac{15}{4}$ pounds. $\frac{1}{2}$ pound is equal to $\frac{2}{4}$ pound. $\frac{15}{4} + \frac{2}{4} = \frac{17}{4} = 4\frac{1}{4} = 4.25$ lb, which is just enough to win a free movie coupon.

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24. Last week, several gas stations in a neighborhood all charged the same price for a gallon of gas. The table below shows how much gas prices have changed from last week to this week.

Gas Station	Gas and Go	Samson Gas	Star Gas	Corner Store	Tip Top Shop
Change from last week (in cents)	−6.6	5.8	$-6\frac{3}{4}$	$\frac{27}{5}$	$-5\frac{5}{8}$

a. Order the numbers in the table from least to greatest.

$-6\frac{3}{4}$, −6.6, $-5\frac{5}{8}$, $\frac{27}{5}$, 5.8

b. Which gas station has the cheapest gas this week? Star Gas

c. **Critical Thinking** Which gas station changed their price the least this week?

Corner Store

H.O.T. FOCUS ON HIGHER ORDER THINKING

25. **Analyze Relationships** Explain how you would order from least to greatest three numbers that include a positive number, a negative number, and zero.

Negative numbers are less than zero and positive numbers are greater than zero, so the order would be negative, zero, positive.

26. **Critique Reasoning** Luke is making pancakes. The recipe calls for 0.5 quart of milk and 2.5 cups of flour. He has $\frac{3}{8}$ quart of milk and $\frac{18}{8}$ cups of flour. Luke makes the recipe with the milk and flour that he has. Explain his error.

Luke does not have enough of either ingredient. He has $\frac{3}{8}$ of a quart of milk, or 0.375. $0.375 < 0.5$. He has $\frac{18}{8}$ cups of flour, or 2.25. $2.25 < 2.5$.

27. **Communicate Mathematical Ideas** If you know the order from least to greatest of 5 negative rational numbers, how can you use that information to order the absolute values of those numbers from least to greatest? Explain.

The order of the absolute values will be the reverse of the order of the negative rational numbers. The least negative number will be the farthest from 0 on a number line. The greatest negative number will be closest to 0 on a number line.

Work Area

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EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Have students use the given information to find the Mystery Number. After students solve the number puzzle, ask them to share the methods they used to identify the number. Then encourage students to write a similar number puzzle and challenge other students to solve it.

The Mystery Number is -0.15 or $-\frac{3}{20}$.

Mystery Number

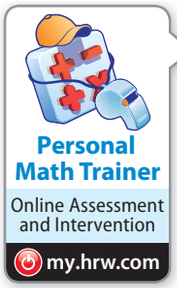
1. The absolute value of the mystery number is less than $\frac{1}{2}$ but greater than $\frac{1}{10}$.
2. The mystery number is to the left of 0 on the number line.
3. When written as a decimal, the mystery number requires 2 places to the right of the decimal point.
4. As a fraction in simplest form, the denominator is a multiple of 10 and the numerator is an odd number.

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention



Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Differentiated Instruction

- Reteach worksheets
- Reading Strategies ELL
- Success for English Learners ELL

Differentiated Instruction

- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE

Additional Resources

Assessment Resources includes:

- Leveled Module Quizzes

MODULE QUIZ

Ready to Go On?

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2.1 Classifying Rational Numbers

1. Five friends divide three bags of apples equally between them. Write the division represented in this situation as a fraction.

$\frac{3}{5}$

Write each rational number as $\frac{a}{b}$.

2. $5\frac{1}{6}$ $\frac{31}{6}$

3. -12 $-\frac{12}{1}$

Determine if each number is a whole number, integer, or rational number. Include all sets to which each number belongs.

4. -12 integer and rational number

5. $\frac{7}{8}$ rational number

2.2 Identifying Opposites and Absolute Value of Rational Numbers

6. Graph -3, $1\frac{3}{4}$, -0.5, and 3 on the number line.

7. Find the opposite of $\frac{1}{3}$ and $-\frac{7}{12}$ $-\frac{1}{3}$; $\frac{7}{12}$

8. Find the absolute value of 9.8 and $-\frac{10}{3}$ 9.8; $\frac{10}{3}$

2.3 Comparing and Ordering Rational Numbers

9. Over the last week, the daily low temperatures in degrees Fahrenheit have been -4, 6.2, $18\frac{1}{2}$, -5.9, 21, $-\frac{1}{4}$, and 1.75. List these numbers in order from greatest to least.

21, $18\frac{1}{2}$, 6.2, 1.75, $-\frac{1}{4}$, -4, -5.9

ESSENTIAL QUESTION

10. How can you solve problems by ordering rational numbers from least to greatest?

Convert so that all numbers are in the same form.
Order them as they appear on the number line.

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Module 2 49

Texas Essential Knowledge and Skills

Lesson	Exercises	TEKS
2.1	1–5	**6.2.A, 6.2.E**
2.2	6–10	**6.2.B**
2.3	11–12	**6.2.D**

Texas Test Prep

Texas Testing Tip Some items are called context-based items, which means the student has to examine each answer choice in order to determine the correct answer.

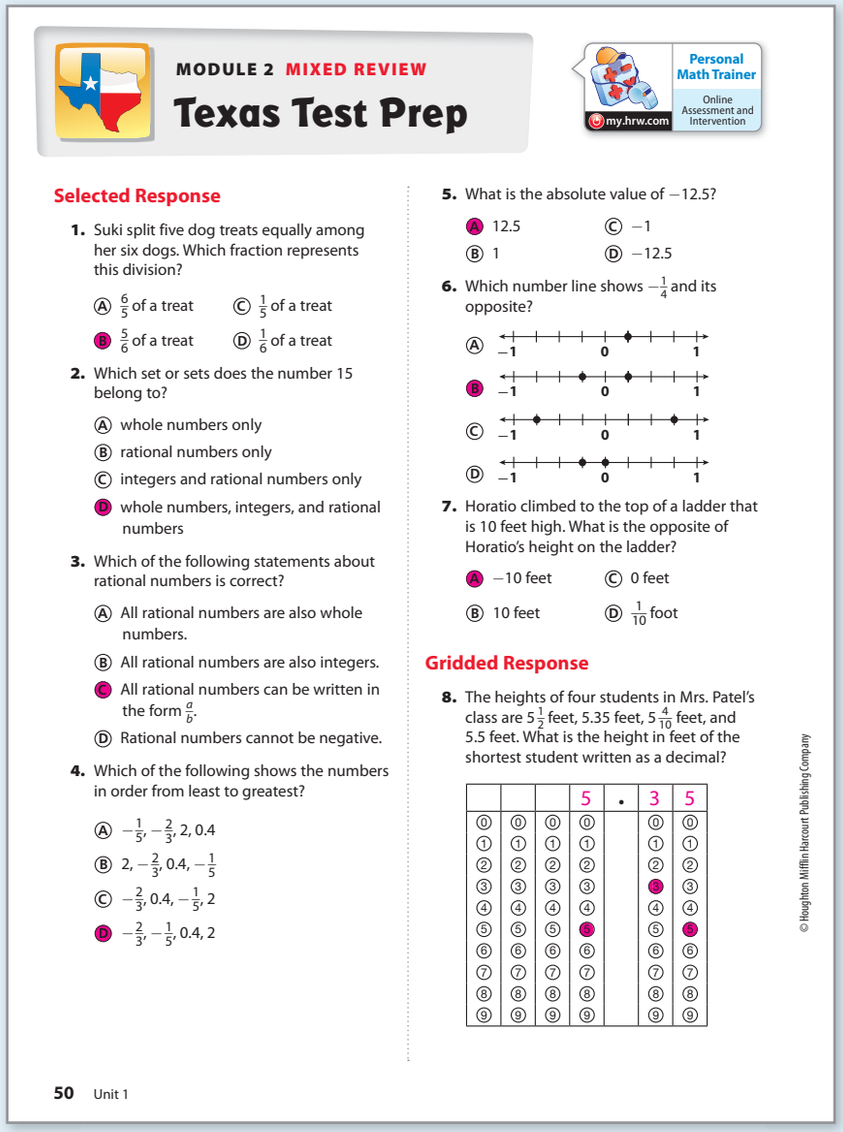
Item 2 It may be helpful for students to first sketch a Venn diagram to model the relationship between number sets. This will help them see that D is the correct answer.

Item 7 Because each answer choice contains the same numbers, students can plot the numbers on a number line and then read them from left to right to see that D is the correct answer.

Avoid Common Errors

Item 2 Students may accidentally miss the word *only* in these answer choices. Caution them to read carefully.

Item 6 To avoid errors reading the scale on the number lines, point out that the space between 0 and 1 is divided into 4 equal sections.



MODULE 2 MIXED REVIEW

Texas Test Prep

Personal Math Trainer
Online Assessment and Intervention
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Selected Response

1. Suki split five dog treats equally among her six dogs. Which fraction represents this division?
   - Ⓐ $\frac{6}{5}$ of a treat
   - Ⓒ $\frac{1}{5}$ of a treat
   - Ⓑ $\frac{5}{6}$ of a treat
   - Ⓓ $\frac{1}{6}$ of a treat
2. Which set or sets does the number 15 belong to?
   - Ⓐ whole numbers only
   - Ⓑ rational numbers only
   - Ⓒ integers and rational numbers only
   - Ⓓ whole numbers, integers, and rational numbers
3. Which of the following statements about rational numbers is correct?
   - Ⓐ All rational numbers are also whole numbers.
   - Ⓑ All rational numbers are also integers.
   - Ⓒ All rational numbers can be written in the form $\frac{a}{b}$.
   - Ⓓ Rational numbers cannot be negative.
4. Which of the following shows the numbers in order from least to greatest?
   - Ⓐ $-\frac{1}{5}, -\frac{2}{3}, 2, 0.4$
   - Ⓑ $2, -\frac{2}{3}, 0.4, -\frac{1}{5}$
   - Ⓒ $-\frac{2}{3}, 0.4, -\frac{1}{5}, 2$
   - Ⓓ $-\frac{2}{3}, -\frac{1}{5}, 0.4, 2$
5. What is the absolute value of −12.5?
   - Ⓐ 12.5
   - Ⓒ −1
   - Ⓑ 1
   - Ⓓ −12.5
6. Which number line shows $-\frac{1}{4}$ and its opposite?
   - Ⓐ
   - Ⓑ
   - Ⓒ
   - Ⓓ
7. Horatio climbed to the top of a ladder that is 10 feet high. What is the opposite of Horatio's height on the ladder?
   - Ⓐ −10 feet
   - Ⓒ 0 feet
   - Ⓑ 10 feet
   - Ⓓ $\frac{1}{10}$ foot

Gridded Response

8. The heights of four students in Mrs. Patel's class are $5\frac{1}{2}$ feet, 5.35 feet, $5\frac{4}{10}$ feet, and 5.5 feet. What is the height in feet of the shortest student written as a decimal?

			5	.	3	5
0	0	0	0		0	0
1	1	1	1		1	1
2	2	2	2		2	2
3	3	3	3		●	3
4	4	4	4		4	4
5	5	5	●		5	●
6	6	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	9		9	9

50 Unit 1

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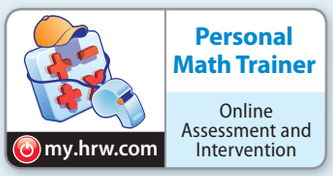
Texas Essential Knowledge and Skills

Items	Grade 6 TEKS	Mathematical Process TEKS
1	6.2.E	6.1.A
2	6.2.A	6.1.F
3	6.2.A	6.1.F
4	6.2.B	6.1.F
5	6.2.B	6.1.F
6*	6.2.B	6.1.D, 6.1.E
7	6.2.D	6.1.D
8	6.2.D, 6.2.E	6.1.A

* Item integrates mixed review concepts from previous modules or a previous course.

Study Guide Review

Additional Resources



Assessment Resources

- Leveled Unit Tests: A, B, C, D
- Performance Assessment

Vocabulary Development

Integrating the ELPS

Encourage English learners to refer to their notes and the illustrated, bilingual glossary as they review the unit content.

ELPS **c.4.E** Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

MODULE 1 Integers

TEKS **6.2.B. 6.2.C**

Key Concepts

- Integers are positive and negative whole numbers. *(Lesson 1.1)*
- Inequality symbols include $>$, or "greater than," and $<$, or "less than." *(Lesson 1.2)*
- The absolute value of a number is always positive, since it is the number's distance from 0. *(Lesson 1.3)*

MODULE 2 Rational Numbers

TEKS **6.2.A, 6.2.B, 6.2.D. 6.2.E**

Key Concepts

- A rational number is any number that can be written as $\frac{a}{b}$. *(Lesson 2.1)*
- The opposite of a rational number is the number the same distance from 0 on the number line but on the opposite side of 0. *(Lesson 2.2)*
- To compare and order rational numbers, convert them to decimals or fractions. *(Lesson 2.3)*

UNIT 1

Study Guide Review

MODULE 1 Integers

ESSENTIAL QUESTION

How can you use integers to solve real-world problems?

Key Vocabulary
- absolute value *(valor absoluto)*
- inequality *(desigualdad)*
- integers *(entero)*
- negative numbers *(número negativo)*
- opposites *(opuestos)*
- positive numbers *(número positivo)*

EXAMPLE 1

James recorded the temperature at noon in Fairbanks, Alaska, over a week in January.

Day	Mon	Tues	Wed	Thurs	Fri
Temperature	3	2	7	−3	−1

Graph the temperatures on the number line, and then list the numbers in order from least to greatest.

Graph the temperatures on the number line.

Read from left to right to list the temperatures in order from least to greatest.

The temperatures listed from least to greatest are −3, −1, 2, 3, 7.

EXAMPLE 2

Graph the following numbers on the number line. Then use the number line to find each absolute value.

−4 0 2 −1

|−4| = 4 |0| = 0

|2| = 2 |−1| = 1

EXERCISES

1. Graph each number on the number line. *(Lesson 1.1)*

7, −2, 5, 1, −1

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Write the opposite of each number . *(Lesson 1.1)*

2. 8 −8 **3.** −3 3

List the numbers from least to greatest. *(Lesson 1.2)*

4. 4, 0, −2, 3 −2, 0, 3, 4

5. −3, −5, 2, −2 −5, −3, −2, 2

Use a number line to help you compare the numbers. Use < or >. *(Lesson 1.2)*

6. 4 > 1 **7.** −2 < 2

8. −3 > −5 **9.** −7 < 2

Find each absolute value. *(Lesson 1.3)*

10. |6| 6 **11.** |−2| 2

MODULE 2 Rational Numbers

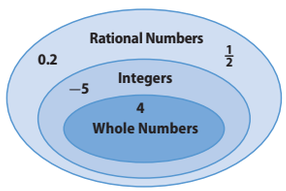
ESSENTIAL QUESTION

How can you use rational numbers to solve real-world problems?

Key Vocabulary
- rational number *(número racional)*
- Venn digram *(diagrama de Venn)*

EXAMPLE 1

Use the Venn diagram to determine in which set or sets each number belongs.



A. $\frac{1}{2}$ The number $\frac{1}{2}$ belongs in the sets of rational numbers.

B. −5 The number −5 belongs in the sets of integers and rational numbers.

C. 4 The number 4 belongs in the set of whole numbers, integers, and rational numbers.

D. 0.2 The number 0.2 belongs in the set of rational numbers.

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CAREERS IN MATH
For more information about careers in mathematics as well as various mathematics appreciation topics, visit the American Mathematical Society at www.ams.org

Unit 1 Performance Tasks

The Performance Tasks provide students with the opportunity to apply concepts from this unit in real-world problem situations.

CAREERS IN MATH

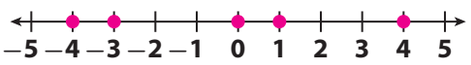
Climatologist In Performance Task Item 1, students can see how a climatologist uses mathematics on the job.

SCORING GUIDES FOR PERFORMANCE TASKS

1. MATHEMATICAL PROCESSES TEKS **6.1.A, 6.1.D, 6.2.D**

Task	Possible Points (Total: 6)
a	**2 points** for correct list: 1920, 1900, 1910, 1940, 1930
b	**1 point** for correct year: 1930 **1 point** for explanation: This was the year when the ring was widest, which means that year had the greatest average temperature.
c	**1 point** for correct year: 1920 **1 point** for explanation: This was the year when the ring was narrowest, which means that year had the least average temperature.

2. MATHEMATICAL PROCESSES TEKS **6.1.A, 6.2.C**

Task	Possible Points (Total: 6)
a	**1 point** for listing integers: –4, –3, and 4 **1 point** for explanation: I will use positive and negative integers because some floors are below ground level, and some floors are above ground level. Zero is ground level.
b	**1 point** for graphing all values: 
c	**1 point** for listing values in order: –4, –3, 0, 1, 4
d	**1 point** for correct number of flights of stairs: 8 flights **1 point** for explanation: Starting at –4, I count integers to the right until I reach 4, for a total of 8, so that means that starting at the fourth floor below ground level, Gala must climb 8 flights of stairs to get to the fourth floor above ground level.

EXAMPLE 2

A. Order $\frac{1}{10}$, 0.9, 0.2, $\frac{3}{5}$, and 0.35 from least to greatest.

Write the fractions as equivalent decimals. $\frac{1}{10} = 0.1$ $\frac{3}{5} = 0.6$

Use the number line to write the decimals in order.

$0.1 < 0.2 < 0.35 < 0.6 < 0.9$

The numbers in order from least to greatest are $\frac{1}{10}$, 0.2, 0.35, $\frac{3}{5}$, 0.9.

B. Order $\frac{2}{5}$, 0.2, and $\frac{4}{15}$ from greatest to least.

Write the decimal as an equivalent fraction. $0.2 = \frac{2}{10} = \frac{1}{5}$

Find equivalent fractions with 15 as the common denominator.

$\frac{2 \times 3}{5 \times 3} = \frac{6}{15}$ $\frac{1 \times 3}{5 \times 3} = \frac{3}{15}$ $\frac{4}{15} = \frac{4}{15}$

Order fractions with common denominators by comparing the numerators.

$6 > 4 > 3$ $\frac{6}{15} > \frac{4}{15} > \frac{3}{15}$

The numbers in order from greatest to least are, $\frac{2}{5}$, $\frac{4}{15}$, and 0.2.

EXERCISES

Classify each number by indicating in which set or sets it belongs. (Lesson 2.1)

1. 8 whole numbers, integers, rational numbers

2. 0.25 rational numbers

Find the absolute value of each rational number. (Lesson 2.2)

3. $|3.7|$ 3.7 **4.** $\left|-\frac{2}{3}\right|$ $\frac{2}{3}$

Graph each set of numbers on the number line and order the numbers from greatest to least. (Lesson 2.1, 2.3)

5. $-0.5, -1, -\frac{1}{4}, 0$

$0, -\frac{1}{4}, -0.5, -1$

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Unit 1 Performance Tasks

1. **CAREERS IN MATH** **Climatologist** Each year a tree is alive, it adds a layer of growth, called a tree ring, between its core and its bark. A climatologist measures the width of tree rings of a particular tree for different years:

Year	1900	1910	1920	1930	1940
Width of ring (in mm)	$\frac{14}{25}$	$\frac{29}{50}$	$\frac{53}{100}$	$\frac{13}{20}$	$\frac{3}{5}$

The average temperature during the growing season is directly related to the width of the ring, with a greater width corresponding to a higher average temperature.

a. List the years in order of increasing ring width.

1920, 1900, 1910, 1940, 1930

b. Which year was hottest? How do you know?

1930; its ring is the widest.

c. Which year was coldest? How do you know?

1920; its ring is the narrowest.

2. A parking garage has floors above and below ground level. For a scavenger hunt, Gaia's friends are given a list of objects they need to find on the third and fourth level below ground, the first and fourth level above ground, and ground level.

a. If ground level is 0 and the first level above ground is 1, which integers can you use to represent the other levels where objects are hidden? Explain your reasoning.

−4, −3, and 4; if positive 1 represents the first level above ground, negative numbers are a good choice to represent levels below ground.

b. Graph the set of numbers on the number line.

c. Gaia wants to start at the lowest level and work her way up. List the levels in the order that Gaia will search them.

−4, −3, 0, 1, 4

d. If she takes the stairs, how many flights of stairs will she have to climb? How do you know?

8 flights; starting at −4 and moving to 4, you pass a total of 8 integers on the number line.

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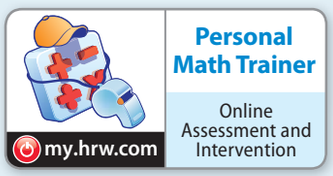
MIXED REVIEW

Texas Test Prep

Additional Resources



Assessment Resources

- Leveled Unit Tests: A, B, C, D
- Performance Assessment

Texas Testing Tip Students can create a horizontal place value chart to help with the placement of answers into the grids.

Item 13 A place value chart will help students to correctly place the first digit of the answer into the hundreds place and the last digit of the answer into the tenths place.

Item 14 Students often misplace whole-number answers within the grid—either all the way to the left or all the way to the right. If they create a place value chart, they are less likely to make this mistake.

Avoid Common Errors

Item 4 Some students will ignore the negative and choose Albany because it appears to be the smallest number. Remind students that in this context, negative numbers represent colder temperatures as their absolute values get larger, and encourage them to sketch a number line if they need a reminder.

Item 8 Because the number in the problem does not include a negative sign, some students may select answer choice A. Remind students that the language and context of the problem can represent a negative quantity even when a negative sign is not used.

Texas Essential Knowledge and Skills

Items	Grade 6 TEKS	Mathematical Process TEKS
1	6.2.B	6.1.C
2	6.2.B	6.1.A
3	6.2.B	6.1.C
4	6.2.B	6.1.A
5	6.2.C	6.1.C
6	6.2.E	6.1.A
7	6.2.A	6.1.F
8	6.2.B	6.1.A
9	6.2.B	6.1.D
10*	4.3.G, 6.2.D	6.1.C
11	6.2.B	6.1.D
12	6.2.C	6.1.C
13	6.2.B	6.1.A
14	6.2.B	6.1.A

* Item integrates mixed review concepts from previous modules or a previous course.

UNIT 1 MIXED REVIEW

Texas Test Prep

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Selected Response

1. What is the opposite of −9?

Ⓐ 9
Ⓑ $-\frac{1}{9}$
Ⓒ 0
Ⓓ $\frac{1}{9}$

2. Kyle is currently 60 feet above sea level. Which correctly describes the opposite of Kyle's elevation?

Ⓐ 60 feet below sea level
Ⓑ 60 feet above sea level
Ⓒ 6 feet below sea level
Ⓓ At sea level

3. What is the absolute value of 27?

Ⓐ −27
Ⓑ 0
Ⓒ 3
Ⓓ 27

4. In Albany it is −4°F, in Chicago it is −14°F, in Minneapolis it is −11°F, and in Toronto it is −13°F. In which city is it the coldest?

Ⓐ Albany
Ⓑ Chicago
Ⓒ Minneapolis
Ⓓ Toronto

5. Which shows the integers in order from greatest to least?

Ⓐ 18, 4, 3, −2, −15
Ⓑ −2, 3, 4, −15, 18
Ⓒ −15, −2, 3, 4, 18
Ⓓ 18, −15, 4, 3, −2

6. Joanna split three pitchers of water equally among her eight plants. What fraction of a pitcher did each plant get?

Ⓐ $\frac{1}{8}$ of a pitcher
Ⓑ $\frac{1}{3}$ of a pitcher
Ⓒ $\frac{3}{8}$ of a pitcher
Ⓓ $\frac{8}{3}$ of a pitcher

7. Which set or sets does the number −22 belong to?

Ⓐ Whole numbers only
Ⓑ Rational numbers only
Ⓒ Integers and rational numbers only
Ⓓ Whole numbers, integers, and rational numbers

8. Carlos swam to the bottom of a pool that is 12 feet deep. What is the opposite of Carlos's elevation relative to the surface?

Ⓐ −12 feet
Ⓑ 0 feet
Ⓒ 12 feet
Ⓓ $\frac{1}{12}$ foot

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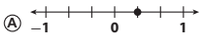
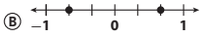
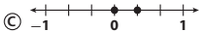
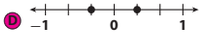
9. Which number line shows $\frac{1}{3}$ and its opposite?

Ⓐ 

Ⓑ 

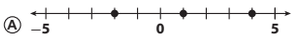
Ⓒ 

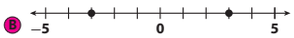
Ⓓ 

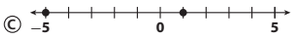
10. Which of the following shows the numbers in order from least to greatest?

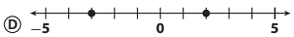
Ⓐ $-\frac{2}{3}, -\frac{3}{4}$, 0.7, 0
Ⓑ 0.7, 0, $-\frac{2}{3}, -\frac{3}{4}$,
Ⓒ $-\frac{2}{3}, -\frac{3}{4}$, 0, 0.7
Ⓓ $-\frac{3}{4}, -\frac{2}{3}$, 0, 0.7

11. Which number line shows an integer and its opposite?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Gridded Response

12. Which is the greatest out of $\frac{1}{3}$, −1.2, 0.45, and $-\frac{4}{5}$?

			0	.	4	5
	0	0	●		0	0
	1	1	1		1	1
	2	2	2		2	2
	3	3	3		3	3
	4	4	4		●	4
	5	5	5		5	●
	6	6	6		6	6
	7	7	7		7	7
	8	8	8		8	8
	9	9	9		9	9

13. As part of a research team, Ryanne climbed into a cavern to an elevation of −117.6 feet. What is the absolute value of Ryanne's elevation, in feet?

	1	1	7	.	6	
0	0	0	0		0	0
1	●	●	1		1	1
2	2	2	2		2	2
3	3	3	3		3	3
4	4	4	4		4	4
5	5	5	5		5	5
6	6	6	6		●	6
7	7	7	●		7	7
8	8	8	8		8	8
9	9	9	9		9	9

Hot Tip! **These grids do not allow you to give an answer that is negative. If you get a negative value, you likely made an error. Check your work!**

14. Melvin has a certain number of files on his computer. The opposite of this number is −653. How many files are on Melvin's computer?

	6	5	3	.		
0	0	0	0		0	0
1	1	1	1		1	1
2	2	2	2		2	2
3	3	3	●		3	3
4	4	4	4		4	4
5	5	●	5		5	5
6	●	6	6		6	6
7	7	7	7		7	7
8	8	8	8		8	8
9	9	9	9		9	9

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