Integers

ESSENTIAL QUESTION

How can you use integers to solve real-world problems?

You can represent real-world quantities such as temperatures, elevations, and gains and losses of money with positive and negative integers.

Real-World Video

Integers can be used to describe the value of many things in the real world. The height of a mountain in feet may be a very great integer while the temperature in degrees Celsius at the top of that mountain may be a negative integer.

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Math On the Spot
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Animated Math
Interactively explore key concepts to see how math works.

Personal Math Trainer
Get immediate feedback and help as you work through practice sets.

Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

Intervention	Enrichment

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 4 Compare Whole Numbers
- Skill 5 Order Whole Numbers
- Skill 61 Locate Numbers on a Number Line

Differentiated Instruction
- Challenge worksheets **PRE-AP**

Extend the Math **PRE-AP**
  Lesson Activities in TE

Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.

Compare Whole Numbers

EXAMPLE

3,564 ● 3,528   Compare digits in the thousands place: $3 = 3$
3,564 ● 3,528   Compare digits in the hundreds place: $5 = 5$
3,564 $>$ 3,528   Compare digits in the tens place: $6 > 2$

Compare. Write $<$, $>$, or $=$.

1. 471 $>$ 468
2. 5,005 $<$ 5,050
3. 398 $>$ 389
4. 10,973 $<$ 10,999
5. 8,471 $<$ 9,001
6. 108 $>$ 95

Order Whole Numbers

EXAMPLE

356, 348, 59, 416   Compare digits. Find the greatest number.
356, 348, 59, 416   Find the next greatest number.
356, 348, 59, 416   Find the next greatest number.
356, 348, 59, 416   Find the least number.
$416 > 356 > 348 > 59$ Order the numbers.

Order the numbers from greatest to least.

7. 156, 87, 177, 99
   $177 > 156 > 99 > 87$
8. 591, 589, 603, 600
   $603 > 600 > 591 > 589$
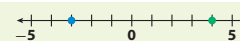9. 2,650, 2,605, 3,056, 2,088
   $3,056 > 2,650 > 2,605 > 2,088$
10. 1,037, 995, 10,415, 1,029
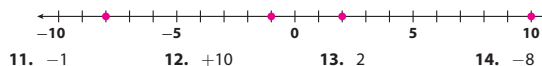 $10,415 > 1,037 > 1,029 > 995$

Locate Numbers on a Number Line

EXAMPLE

Graph $+4$ by starting at 0 and counting 4 units to the right.
Graph -3 by starting at 0 and counting 3 units to the left.

Graph each number on the number line.

11. -1
12. $+10$
13. 2
14. -8

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores integers in an actual sixth-grade classroom.

Professional Development
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Online Teacher Edition
Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

ePlanner
Easily plan your classes and access all your resources online.

Interactive Answers and Solutions
Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated, TEKS-aligned practice tests.

Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The definition and example chart helps students learn the symbols used in this chapter. Explain to students that a symbol is a character that represents a mathematical relationship or operation. To help students understand the concept of symbols, write a few symbols from real life on the board, such the symbol for money or dollar sign ($) and the "at" symbol used in e-mail (@).

Understand Vocabulary

Use the following explanations to help students learn the preview words.

> On a thermometer, if the temperature is above 0, it is written as a **positive number**. If the temperature is below 0, it is written as a **negative number**. For example, if the temperature is 10 degrees below 0, it is written as −10, or minus 10 degrees.

Active Reading

Integrating the ELPS

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

ELPS c.4.D Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary to enhance comprehension of written text.

Additional Resources

Differentiated Instruction

• Reading Strategies **ELL**

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the chart. Write the correct vocabulary word next to the symbol.

Symbol	
<	less than
>	greater than
=	equal
+	plus sign
−	negative sign

Understand Vocabulary

Complete the sentences using the preview words.

1. An __inequality__ is a statement that two quantities are not equal.

2. The set of all whole numbers and their opposites are __integers__.

3. Numbers greater than 0 are __positive numbers__. Numbers less than 0 are __negative numbers__.

Vocabulary

Review Words
✔ equal *(igual)*
✔ greater than *(más que)*
✔ less than *(menos que)*
✔ negative sign *(signo negativo)*
  number line *(recta numérica)*
✔ plus sign *(signo más)*
  symbol *(símbolo)*
  whole number *(número entero)*

Preview Words
  absolute value *(valor absoluto)*
  inequality *(desigualdad)*
  integers *(enteros)*
  negative numbers *(números negativos)*
  opposites *(opuestos)*
  positive numbers *(números positivos)*

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions in this module.

Grades 6–8 TEKS

Before	In this module	After
Students understand whole numbers, fractions, and decimals: • compare and order whole numbers • compare and order fractions • compare and order decimals	Students recognize, order, and perform computations with integers: • identify a number and its opposite • compare and order integers using a number line • find the absolute value of a number	Students will connect whole numbers and integers: • locate, compare, and order integers using a number line • perform operations with integers

Unpacking the TEKS

. . . .

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Texas Essential Knowledge and Skills

Content Focal Areas

TEKS **Number and Operations—6.2**

The student applies mathematical process standards to represent and use rational numbers in a variety of forms.

Integrating the ELPS

ELPS **c.4.F** Use visual and contextual support . . . to read grade-appropriate content area text . . . and develop vocabulary . . . to comprehend increasingly challenging language.

Go online to see a complete unpacking of the TEKS.

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Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 6.2.B

Identify a number, its opposite, and its absolute value.

Key Vocabulary

Integers *(enteros)*
The set of all whole numbers and their opposites.

opposites *(opuestos)*
Two numbers that are equal distance from zero on a number line.
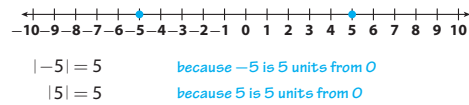
absolute value *(valor absoluto)*
A number's distance from 0 on the number line.

What It Means to You

You will learn see that the absolute value of a number is its distance from 0.

UNPACKING EXAMPLE 6.2.B

Use the number line to determine the absolute values.

$$-10\,-9\,-8\,-7\,-6\,-5\,-4\,-3\,-2\,-1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10$$

$|-5| = 5$ *because −5 is 5 units from 0*

$|5| = 5$ *because 5 is 5 units from 0*

TEKS 6.2.C

Locate, compare, and order integers and rational numbers using a number line.

Key Vocabulary

rational number *(número racional)*
Any number that can be expressed as a ratio of two integers.

What It Means to You

You can use a number line to order rational numbers.

UNPACKING EXAMPLE 6.2.C

At a golf tournament, David scored $+6$, Celia scored -16, and Xavier scored -4. One of these three players was the winner of the tournament. Who won the tournament?

The winner will be the player with the lowest score. Draw a number line and graph each player's score.

$$-18\,-16\,-14\,-12\,-10\ \ -8\ \ -6\ \ -4\ \ -2\ \ 0\ \ 2\ \ 4\ \ 6\ \ 8$$
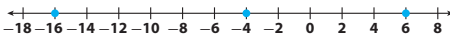
Celia's score, -16, is the farthest to the left, so it is the lowest score. Celia won the tournament.

Visit **my.hrw.com** to see all the **TEKS** unpacked.

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Grade 6 TEKS	Lesson 1.1	Lesson 1.2	Lesson 1.3
TEKS 6.2.B Identify a number, its opposite, and its absolute value.	✓		✓
TEKS 6.2.C Locate, compare, and order integers and rational numbers using a number line.		✓	

LESSON
1.1 Identifying Integers and Their Opposites

Texas Essential Knowledge and Skills

The student is expected to:

TEKS **Number and operations—6.2.B**

Identify a number, its opposite, and its absolute value.

Mathematical Processes

TEKS **6.1.D**

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Engage

ESSENTIAL QUESTION

How do you identify an integer and its opposite? Look for numbers that are the same distance from zero and on opposite sides of zero on the number line; for example, −4 and 4.

Motivate the Lesson

Ask: What is the coldest weather you have ever experienced? Have you ever experienced a temperature that is below zero? How do you write a temperature that is below zero? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1

Focus on Modeling 🔲 Mathematical Processes
Point out to students that the number line is presented horizontally, but for elevation it is useful to think of it vertically. You may want to draw a vertical number line on the board and label the various locations presented in the table on the vertical number line.

Explain

EXPLORE ACTIVITY 2

Connect Vocabulary ELL 🔲 ELPS c.1.A
To help students understand the concept of **opposite** in math and in other contexts, make a list with students of pairs of opposites, such as hot and cold, black and white, up and down, left and right. Clarify that left and right is used in the math concept of opposite with negative numbers to the left of 0 and positive numbers to the right. Zero is its own opposite.

Questioning Strategies 🔲 Mathematical Processes
• Does every integer have an opposite? Explain. Yes, zero is its own opposite. For all other integers, the opposite has a different sign.

• How does a number line help you understand what the opposite of an integer is? I can visually see that 4 and −4 are the same distance from zero.

Connect to Daily Life
Explain that bank statements record amounts of money being withdrawn or spent as negative amounts and amounts of money being deposited as positive amounts.

Talk About It
Check for Understanding
Ask: How do you find the opposite of an integer? Look for the integer that is the same distance from 0 but on the other side of zero.

Identifying Integers and Their Opposites

? ESSENTIAL QUESTION

How do you identify an integer and its opposite?

EXPLORE ACTIVITY 1 (Real World) **TEKS 6.2.B**

Positive and Negative Numbers

Positive numbers are numbers greater than 0. Positive numbers can be written with or without a plus sign; for example, 3 is the same as +3. **Negative numbers** are numbers less than 0. Negative numbers must always be written with a negative sign.

> The number 0 is neither positive nor negative.

−5 −4 −3 −2 −1 0 1 2 3 4 5

Negative integers   Positive integers
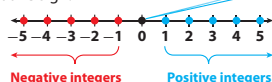
The elevation of a location describes its height above or below sea level, which has elevation 0. Elevations below sea level are represented by negative numbers, and elevations above sea level are represented by positive numbers.

A The table shows the elevations of several locations in a state park. Graph the locations on the number line according to their elevations.

Location	Little Butte A	Cradle Creek B	Dinosaur Valley C	Mesa Ridge D	Juniper Trail E
Elevation (ft)	5	−5	−9	8	−3

C   B   E   A   D
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10
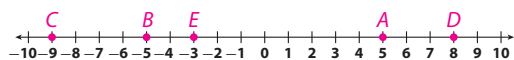
B What point on the number line represents sea level? __0__

C Which location is closest to sea level? How do you know?
Juniper Trail; its elev. is closest to 0 on the number line.

D Which two locations are the same distance from sea level? Are these locations above or below sea level?
Little Butte (above) and Cradle Creek (below)

E Which location has the least elevation? How do you know?
Dinosaur Valley; its elev. is farthest left on the number line.

Lesson 1.1 **7**

EXPLORE ACTIVITY (cont'd)

Reflect

1. **Analyze Relationships** Morning Glory Stream is 7 feet below sea level. What number represents the elevation of Morning Glory Stream?
   −7

2. **Multiple Representations** Explain how to graph the elevation of Morning Glory Stream on a number line.
   Graph a point 7 units to the left of 0 on the number line.

EXPLORE ACTIVITY 2 **TEKS 6.2.B**

Opposites

Two numbers are **opposites** if, on a number line, they are the same distance from 0 but on different sides of 0. For example, 5 and −5 are opposites. 0 is its own opposite.

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

Integers are the set of all whole numbers and their opposites.
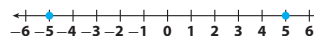
> Remember, the set of whole numbers is 0, 1, 2, 3, 4, 5, 6, …

On graph paper, use a ruler or straightedge to draw a number line. Label the number line with each integer from −10 to 10. Fold your number line in half so that the crease goes through 0. Numbers that line up after folding the number line are opposites.

A Use your number line to find the opposites of 7, −6, 1, and 9. __−7; 6; −1; −9__

B How does your number line show that 0 is its own opposite?
The crease goes through 0, so 0 lines up with itself.

C What is the opposite of the opposite of 3? __3__

Reflect

3. **Justify Reasoning** Explain how your number line shows that 8 and −8 are opposites.
   8 and −8 are the same distance from 0 but on different sides of 0.

4. **Multiple Representations** Explain how to use your number line to find the opposite of the opposite of −6.
   Fold the number line in half at 0. −6 lines up with 6 so 6 is the opposite of −6 and −6 is the opposite of 6. So −6 is the opposite of the opposite of −6.

8 Unit 1

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.D,** which calls for students to "communicate mathematical ideas … using multiple representations, including symbols, … graphs, and language … as appropriate." In each Explore Activity and Example, students use number lines to represent the integers and opposites that are described with language and/or numbers with or without negative symbols. In this way, students are able to make the connections between and become fluent in using the different representations of integers and their opposites.

Math Background

The opposite of any positive number is negative, and the opposite of any negative number is positive. The sum of a number and its opposite is zero, which is neither positive nor negative.

An integer's distance from zero is said to be non-negative instead of positive. When a distance measurement includes a negative symbol, the symbol describes the direction rather than the distance.

EXAMPLE 1

Questioning Strategies 🔲 Mathematical Processes

- Is the opposite of a temperature always colder? Explain. No, because if the temperature is negative, say −5°, then the opposite would be 5°, which would be warmer.

- Is the opposite of an opposite always the number you started with? Give an example. Yes. If you start at 3, the opposite is −3, then the opposite of −3 is 3.

Engage with the Whiteboard

Have students take turns graphing an integer and then have another student graph the integer's opposite on the number line.

Focus on Patterns 🔲 Mathematical Processes

Elicit from students that when finding the opposite of the opposite of a positive number, the pattern of the signs in the steps is +, −, +. When finding the opposite of the opposite of a negative number, the pattern of the signs in the steps is −, +, −.

YOUR TURN

Avoid Common Errors

If students seem to get lost with the notation "the opposite of the opposite of," suggest that they work backward through the sentence. First they find the opposite of 6, which is −6. Then they find the opposite of −6.

Elaborate

· ·

Talk About It
Summarize the Lesson

Ask: How do you find the opposite of an integer? The opposite of an integer is the integer the same distance from zero on the other side of 0. If the integer is 5, then the opposite is −5. If the integer is −3, then the opposite is 3.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–4, you may want to have students take turns graphing an integer and then have another student graph the integer's opposite on the number lines.

Avoid Common Errors

Exercise 1 Remind students to label the points they graph on the number line carefully, so it is clear which point they intend as the answer.

Exercise 9 Remind students that zero is its own opposite.

Talk About It
Check for Understanding

Ask: I am thinking of a number. The opposite of my number is a distance of 8 units from 0. Do you know what my number is? No, because both 8 and −8 are a distance of 8 units from 0. It could be either 8 or −8.

Integers and Opposites on a Number Line

Positive and negative numbers can be used to represent real-world quantities. For example, 3 can represent a temperature that is 3 °F above 0. −3 can represent a temperature that is 3 °F below 0. Both 3 and −3 are 3 units from 0.

EXAMPLE 1 · Real World

TEKS 6.2.B

Sandy kept track of the weekly low temperature in her town for several weeks. The table shows the low temperature in °F for each week.

Week	Week 1	Week 2	Week 3	Week 4
Temperature (°F)	−1	3	−4	2

A Graph the temperature from Week 3 and its opposite on a number line. What do the numbers represent?

STEP 1 Graph the value from Week 3 on the number line.
The value from Week 3 is −4.
Graph a point 4 units below 0.

STEP 2 Graph the opposite of −4.
Graph a point 4 units above 0.

The opposite of −4 is 4.

−4 represents a temperature that is 4 °F below 0 and 4 represents a temperature that is 4 °F above 0.

B The value for Week 5 is the opposite of the opposite of the value from Week 1. What was the high temperature in Week 5?

STEP 1 Graph the value from Week 1 on the number line.
The value from Week 1 is −1.

STEP 2 Graph the opposite of −1.
The opposite of −1 is 1.

STEP 3 Graph the opposite of 1.
The opposite of 1 is −1.

The opposite of the opposite of −1 is −1.
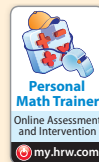The high temperature in Week 5 was −1 °F.

Reflect

5. **Analyze Relationships** Explain how you can find the opposite of the opposite of any number without using a number line.

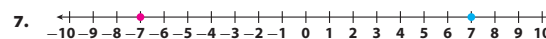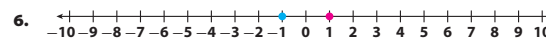**The opposite of the opposite of a number is the number itself.**
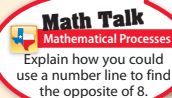
My Notes

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YOUR TURN

Graph the opposite of the number shown on each number line.

6.

7.

Write the opposite of each number.

8. 10 _____ **−10**   **9.** −5 _____ **5**   **10.** 0 _____ **0**

11. What is the opposite of the opposite of 6? _____ **6**

Math Talk
Mathematical Processes
Explain how you could use a number line to find the opposite of 8.
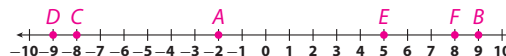
Math Talk anno: First graph a point 8 units to the right of 0. Then graph a point the same distance to the left of 0. That point will be at −8.

Guided Practice

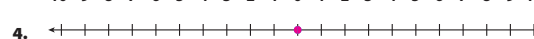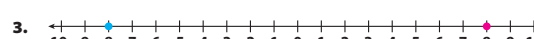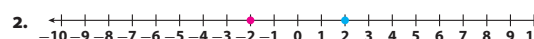**1.** Graph and label the following points on the number line. (Explore Activity 1)

   a. −2   **b.** 9   **c.** −8   **d.** −9   **e.** 5   **f.** 8

Graph the opposite of the number shown on each number line. (Explore Activity 2 and Example 1)

2.

3.

4.

Write the opposite of each number. (Explore Activity 2 and Example 1)

5. 4 _____ **−4**   **6.** −11 _____ **11**   **7.** 3 _____ **−3**

8. −3 _____ **3**   **9.** 0 _____ **0**   **10.** 22 _____ **−22**

? ESSENTIAL QUESTION CHECK-IN

11. Given an integer, how do you find its opposite?

Find the integer that is the same distance from 0 but on the other side of 0.

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DIFFERENTIATE INSTRUCTION

World History

The concept of negative numbers can be traced to Hindu mathematicians. They used negative numbers to represent debts, as we do today, and formulated rules for the arithmetic of integers. Their ideas were acquired by Arab mathematicians, who passed the ideas on to European scientists over time.

Manipulatives

For Explore Activity 2, some students have difficulty labeling a number line and folding it so the opposite integers line up. It may be helpful to give them printed number lines with a vertical dashed line through zero.

Additional Resources

Differentiated Instruction includes:
- Reading Strategies
- Success for English Learners **ELL**
- Reteach
- Challenge **PRE-AP**

1.1 LESSON QUIZ

TEKS 6.2.B

Sara keeps a record of the money that she deposits and withdraws from her account each week.

Week	1	2	3
Account entry ($)	$4	$10	−$8

1. Which week(s) does Sara have a negative entry in her account?

2. Graph each value and its opposite on a number line.

3. Which week's entry was the closest to zero?

4. For Week 4, Sara's entry is the opposite of the opposite of her entry on Week 1. What is her Week 4 entry?

Lesson Quiz available online

my.hrw.com

Answers

1. Week 3

2.

3. Week 1

4. $4

Evaluate

GUIDED AND INDEPENDENT PRACTICE

TEKS 6.2.B

Concepts & Skills	Practice
Explore Activity 1 Positive and Negative Numbers	Exercises 1, 12, 23, 24
Explore Activity 2 Opposites	Exercises 2–10, 12, 13, 15, 18, 19–24
Example 1 Integers and Opposites on a Number Line	Exercises 2–10, 14, 16, 17, 20–23

| Exercise	Depth of Knowledge (D.O.K.)		**TEKS** Mathematical Processes
12	**2** Skills/Concepts		**1.A** Everyday life
13–18	**1** Recall of Information		**1.C** Select tools
19–23	**2** Skills/Concepts		**1.C** Select tools
24	**3** Strategic Thinking	H.O.T.	**1.A** Everyday life
25	**3** Strategic Thinking	H.O.T.	**1.G** Explain and justify arguments
26	**3** Strategic Thinking	H.O.T.	**1.F** Analyze relationships
27	**3** Strategic Thinking	H.O.T.	**1.G** Explain and justify arguments
28	**3** Strategic Thinking	H.O.T.	**1.C** Select tools

Additional Resources

Differentiated Instruction includes:

• Leveled Practice Worksheets

1.1 Independent Practice

TEKS 6.2.B

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com
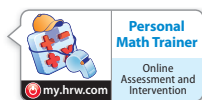
12. Chemistry Atoms normally have an electrical charge of 0. Certain conditions, such as static, can cause atoms to have a positive or a negative charge. Atoms with a positive or negative charge are called *ions*.

Ion	A	B	C	D	E
Charge	−3	+1	−2	+3	−1

a. Which ions have a negative charge?

A, C, E

b. Which ions have charges that are opposites?

A and D; B and E

c. Which ion's charge is not the opposite of another ion's charge?

C

Name the integer that meets the given description.

13. the opposite of −17   17

14. 4 units left of 0   −4

15. the opposite of the opposite of 2   2

16. 15 units right of 0   15

17. 12 units right of 0   12

18. the opposite of −19   19

19. Analyze Relationships Several wrestlers are trying to lose weight for a competition. Their change in weight since last week is shown in the chart.

Wrestler	Tino	Victor	Ramsey	Baxter	Luis
Weight Change (in pounds)	−2	6	2	5	−5

a. Did Victor lose or gain weight since last week?   gain

b. Which wrestler's weight change is the opposite of Ramsey's?   Tino

c. Which wrestlers have lost weight since last week?   Tino and Luis

d. Frankie's weight change since last week was the opposite of Victor's. What was Frankie's weight change?   −6

e. Frankie's goal last week was to gain weight. Did he meet his goal? Explain.

No; −6 pound change means Frankie lost 6 pounds.

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Find the distance between the given number and its opposite on a number line.

20. 6   12 units

21. −2   4 units

22. 0   0 units

23. −7   14 units

24. What If? Three contestants are competing on a trivia game show. The table shows their scores before the final question.

Contestant	Score Before Final Question
Timothy	−25
Shawna	18
Kaylynn	−14

a. How many points must Shawna earn for her score to be the opposite of Timothy's score before the final question?   7 points

b. Which person's score is closest to 0?   Kaylynn

c. Who do you think is winning the game before the final question? Explain.

Shawna; she is the only player with a positive score.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

25. Communicate Mathematical Ideas Which number is farther from 0 on a number line: −9 or 6? Explain your reasoning.

−9; it is 9 units away from 0 on a number line, and 6 is only 6 units away from 0.

26. Analyze Relationships A number is k units to the left of 0 on the number line. Describe the location of its opposite.

Its opposite is k units to the right of 0 on the number line.

27. Critique Reasoning Roberto says that the opposite of a certain integer is −5. Cindy concludes that the opposite of an integer is always negative. Explain Cindy's error.

Cindy assumed the original integer is always positive. But if the original integer is negative, its opposite will be positive.

28. Multiple Representations Explain how to use a number line to find the opposites of the integers 3 units away from −7.

10, 4; −10 is 3 units to the left of −7 and 10 is the opposite of −10. −4 is 3 units to the right of −7 and 4 is the opposite of −4.

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EXTEND THE MATH   PRE-AP

Activity available online my.hrw.com

Activity The lowest and highest places in the United States are both in California, as shown in the graph. How can you use the graph to find the difference in elevation between the two locations?

If you start at the lowest point, you need to go up 282 ft to sea level and then another 14,495 ft to get to the top of Mt. Whitney. 282 + 14,495 = 14,777.
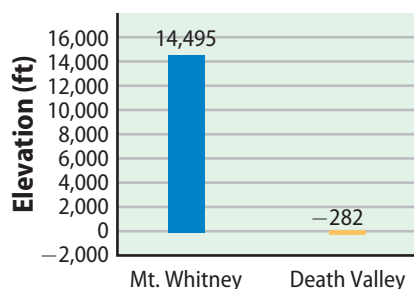
Elevation (ft)

14,495 — Mt. Whitney
−282 — Death Valley

1.2 Comparing and Ordering Integers

ADDITIONAL EXAMPLE 1

Tia's golf scores during her first five days at a golf academy are shown in the table. Graph the scores on a number line, and then list the numbers in order from least to greatest.

Day	Mon	Tue	Wed	Thu	Fri
Score	5	−1	3	2	−3

$$-5 \qquad 0 \qquad 5$$
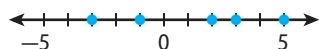
−3, −1, 2, 3, 5

Interactive Whiteboard
Interactive example available online

my.hrw.com

Engage

ESSENTIAL QUESTION

How do you compare and order integers? Graph the integers on a number line, and then read the integers in order from left to right to order them from least to greatest.

Motivate the Lesson
Ask: Which temperature is colder: −20° or −8°? How can you decide? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Focus on Reasoning
Point out to students that teams with negative win/loss records have more losses than wins, while those with positive records have more wins than losses. So when comparing records, if there are more negative than positive records, the league is not very successful, and conversely, if there are more positive then negative records, the league is successful.

Explain

EXAMPLE 1

Talk About It
Check for Understanding
Ask: How does a number line help you order a set of integers? A number line provides a visual representation of the values of the integers in order from least to greatest from left to right.

Questioning Strategies Mathematical Processes
- What is the best score Fred recorded for the week and when does it occur? How do you know? −5 and it occurs on Thursday. −5 has the least value of all the scores recorded and in golf the lowest score, not the highest score, wins the game.
- How do integers change as you move farther left from zero on the number line? They decrease in value.

YOUR TURN

Engage with the Whiteboard
Have students take turns graphing the numbers on the number line and then have another student list the numbers in order from least to greatest.

Talk About It
Check for Understanding
Ask: What do the following changes in stock prices mean: −$5, $4, and $0? A change of −$5 means the stock price fell $5, a change of $4 means the stock price rose $4, and a change of $0 means the stock price did not change.

Comparing and Ordering Integers

TEKS Number and operations—
6.2.C Locate, compare, and order integers … using a number line.

? ESSENTIAL QUESTION

How do you compare and order integers?

EXPLORE ACTIVITY *Real World* **TEKS** 6.2.C

Comparing Positive and Negative Integers

The Westfield soccer league ranks its teams using a number called the "win/loss combined record." A team with more wins than losses will have a positive combined record, and a team with fewer wins than losses will have a negative combined record. The table shows the total win/loss combined record for each team at the end of the season.

Team	Sharks A	Jaguars B	Badgers C	Tigers D	Cougars E	Hawks F	Wolves G
Win/Loss Combined Record	0	4	−4	−6	2	−2	6

A Graph the win/loss combined record for each team on the number line.

D C F A E B G
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10
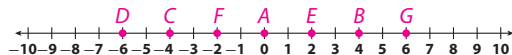
B Which team had the best record in the league? How do you know?

Wolves; their record is farthest right on the number line.

C Which team had the worst record? How do you know?

Tigers; their record is farthest left on the number line.

Reflect

1. **Analyze Relationships** Explain what the data tell you about the win/loss records of the teams in the league.

The Badgers, Tigers, and Hawks lost more games than they won. The Jaguars, Cougars, and Wolves won more games than they lost. The Sharks won the same number of games as they lost.

Lesson 1.2 **13**

Math On the Spot
my.hrw.com

Ordering Positive and Negative Integers

When you read a number line from left to right, the numbers are in order from least to greatest.

EXAMPLE 1 *Real World* **TEKS** 6.2.C

Fred recorded the following golf scores during his first week at the golf academy. In golf, the player with the lowest score wins the game.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Score	4	−2	3	−5	−1	0	−3

Graph Fred's scores on the number line, and then list the numbers in order from least to greatest.

STEP 1 Graph the scores on the number line.

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10
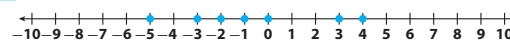
STEP 2 Read from left to right to list the scores in order from least to greatest.

The scores listed from least to greatest are −5, −3, −2, −1, 0, 3, 4.

Math Talk
Mathematical Processes
What day did Fred have his best golf score? How do you know?

Thursday; Fred got his lowest score, −5, on Thursday, and in golf the lowest score is the best.

YOUR TURN

Graph the values in each table on a number line. Then list the numbers in order from greatest to least.
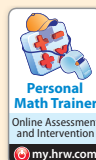
2.
Change in Stock Price ($)					
−5	4	0	−3	−6	2

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

4, 2, 0, −3, −5, −6

3.
Elevation (meters)							
9	−1	−6	2	−10	0	5	8

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

9, 8, 5, 2, 0, −1, −6, −10

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14 Unit 1

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.C,** which calls for students to "select tools … and techniques, including … number sense as appropriate, to solve problems." In the Explore Activity and in both Examples, students use a number line to order and compare integers in real-world contexts, such as ordering rankings, golf scores, and comparing annual precipitation. In this way, students are able to see the integers in terms of their relationship to zero, to each other, and to create statements of numerical order in real-world contexts.

Math Background

The integers consist of the whole numbers, {0, 1, 2, 3, …}, and their opposites, {0, −1, −2, −3, …}. Informally, integers can be defined as the real numbers that can be written without a decimal or fractional component. Students should become adept at visualizing the location of the integers on a number line. In particular, they should be aware of their symmetry about 0. If the number line is folded on itself at 0, each integer is paired with its opposite.
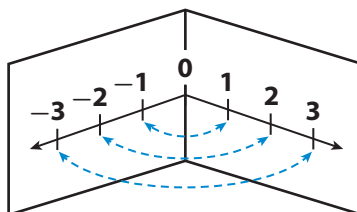
−3 −2 −1 0 1 2 3

EXAMPLE 2

Questioning Strategies 🏴 Mathematical Processes

- Can you always write two different inequality statements to compare two numbers with different values? Explain. Yes, because you can use $>$ to compare the larger number to the smaller number and $<$ to compare the smaller number to the larger number.

- If -1 is the greatest negative integer, is there a least negative integer? Explain. No, the set of negative numbers is infinite, so every negative integer on the number line has an integer of lesser value to its left.

Avoid Common Errors

If students have trouble in determining which inequality sign to use, you may want to remind them that the inequality sign always points to the lesser of two numbers.

YOUR TURN

Avoid Common Errors

When students work with negative numbers, they often think that the number with the greater absolute value is the greater number. You may want to remind them that for negative numbers, the number with the greater absolute value is actually the lesser number because it is farther away from zero in the negative direction.

Elaborate
. . . .

Talk About It
Summarize the Lesson

Ask: How is a number line used to compare and order integers? When the numbers are graphed they are in order of their value. The number line shows the numbers from least to greatest (left to right) and from greatest to least (right to left).

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2, have students use the number line given in Exercise 1 to graph and order the integers.

Avoid Common Errors

Exercise 1 Remind students that the coldest temperature is the least temperature, the one farthest to the left on the number line.

Exercises 2–3 Caution students to pay attention to the signs of the numbers when they create their ordered lists.

Exercise 8 Remind students that when comparing negative integers, the number with the greater absolute value is actually the lesser number because values decrease as one moves left from zero.

Writing Inequalities

An **inequality** is a statement that two quantities are not equal. The symbols $<$ and $>$ are used to write inequalities.

- The symbol $>$ means "is greater than."
- The symbol $<$ means "is less than."
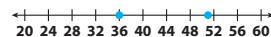
You can use a number line to help write an inequality.

EXAMPLE 2 · Real World · TEKS 6.2.C

A In 2005, Austin, Texas, received 51 inches in annual precipitation. In 2009, the city received 36 inches in annual precipitation. In which year was there more precipitation?

Graph 51 and 36 on the number line.

20 24 28 32 36 40 44 48 52 56 60

- 51 is to the *right* of 36 on the number line.

  This means that 51 is **greater than** 36.

  Write the inequality as $51 > 36$.

- 36 is to the *left* of 51 on the number line.

  This means that 36 is **less than** 51.

  Write the inequality as $36 < 51$.

  There was more precipitation in 2005.

B Write two inequalities to compare -6 and 7.
$-6 < 7; 7 > -6$

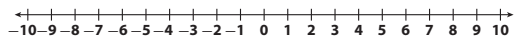**C** Write two inequalities to compare -9 and -4.
$-4 > -9; -9 < -4$

Math Talk
Mathematical Processes
Is there a greatest integer? Is there a greatest negative integer? Explain.

No; for any positive integer, you can find greater positive integers to the right of it on the number line; yes, -1.

YOUR TURN

Compare. Write $>$ or $<$. Use the number line to help you.

4. $-10 \ \boxed{<} \ -2$ **5.** $-6 \ \boxed{<} \ 6$ **6.** $-7 \ \boxed{>} \ -8$

$-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10$

7. Write two inequalities to compare -2 and -18. $-2 > -18; -18 < -2$
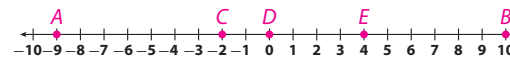
8. Write two inequalities to compare 39 and -39. $-39 < 39; 39 > -39$
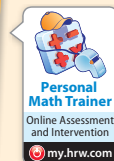
Guided Practice

1a. Graph the temperature for each city on the number line. (Explore Activity)

City	A	B	C	D	E
Temperature (°F)	−9	10	−2	0	4

A   C D   E   B
$-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10$

b. Which city was coldest? _____A_____

c. Which city was warmest? _____B_____

List the numbers in order from least to greatest. (Example 1)

2. $4, -6, 0, 8, -9, 1, -3$
$-9, -6, -3, 0, 1, 4, 8$

3. $-65, 34, 7, -13, 55, 62, -7$
$-65, -13, -7, 7, 34, 55, 62$

4. Write two inequalities to compare -17 and -22. $-17 > -22; -22 < -17$

Compare. Write $<$ or $>$. (Example 2)

5. $-9 \ \boxed{<} \ 2$ **6.** $0 \ \boxed{<} \ 6$ **7.** $3 \ \boxed{>} \ -7$ **8.** $5 \ \boxed{>} \ -10$

9. $-1 \ \boxed{>} \ -3$ **10.** $-8 \ \boxed{<} \ -4$ **11.** $-4 \ \boxed{<} \ 1$ **12.** $-2 \ \boxed{>} \ -6$

13. Compare the temperatures for the following cities. Write $<$ or $>$. (Example 2)

City	Alexandria	Redwood Falls	Grand Marais	Winona	International Falls
Average Temperature in March (°C)	−3	0	−2	2	−4

a. Alexandria and Winona _____$-3 < 2$_____

b. Redwood Falls and International Falls _____$0 > -4$_____

? ESSENTIAL QUESTION CHECK-IN

14. How can you use a number line to compare and order numbers?

The numbers on a number line are in order from least to greatest as you move from left to right.

DIFFERENTIATE INSTRUCTION

Kinesthetic Experience

Have students write the integers being compared on sticky notes and arrange them on a large number line on the board. Ask them to explain why they placed the numbers in the position they did, and encourage them to rearrange the notes if placed incorrectly. Then have students write two inequalities for each comparison they make.

Number Sense

Have students practice comparing numbers without a number line by visualizing them on a number line. For example, **Ask:** *Would -125 be to the left or to the right of -76 on a number line?* Have students challenge one another to tell whether a number is located to the left or right of another number on the number line.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners **ELL**
- Reteach
- Challenge **PRE-AP**

1.2 LESSON QUIZ

🔲 **TEKS** 6.2.C

Use a number line to list the numbers in order from least to greatest.

1. $4, -1, 6, 0, -4, 5, -3$

2. $9, -12, -5, 8, -15, 0, -2$

3. $-38, -16, 45, -24, 71, -63, 10$

4. Write two inequalities to compare -13 and -26.

5. Write two inequalities to compare 0 and -8.

6. At the end of a golf game, Jared's score was -3 and Ned's score was -5. Who won the game? Explain your reasoning.

Lesson Quiz available online

⟳ **my.hrw.com**

Answers
1. $-4, -3, -1, 0, 4, 5, 6$

2. $-15, -12, -5, -2, 0, 8, 9$

3. $-63, -38, -24, -16, 10, 45, 71$

4. $-13 > -26; -26 < -13$

5. $0 > -8; -8 < 0$

6. Ned; In golf, the player with the lowest score wins.

Evaluate

GUIDED AND INDEPENDENT PRACTICE

🔲 **TEKS** 6.2.C

Concepts & Skills	Practice
Explore Activity **Comparing Positive and Negative Integers**	Exercises 1, 15, 19
Example 1 **Ordering Positive and Negative Integers**	Exercises 2–4, 18, 19
Example 2 **Writing Inequalities**	Exercises 5–13, 16, 17, 20–23

Exercise	Depth of Knowledge (D.O.K.)	🔲 TEKS Mathematical Processes
15	**2** Skills/Concepts	**1.D** Multiple representations
16–18	**2** Skills/Concepts	**1.A** Everyday life
19	**2** Skills/Concepts	**1.E** Create and use representations
20–23	**2** Skills/Concepts	**1.A** Everyday life
24	**3** Strategic Thinking **H.O.T.**	**1.F** Analyze relationships
25	**3** Strategic Thinking **H.O.T.**	**1.A** Everyday life
26	**3** Strategic Thinking **H.O.T.**	**1.A** Everyday life
27	**3** Strategic Thinking **H.O.T.**	**1.F** Analyze relationships

Additional Resources
Differentiated Instruction includes:
• Leveled Practice Worksheets

1.2 Independent Practice
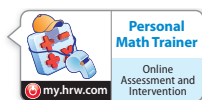
TEKS 6.2.C

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15. **Multiple Representations** A hockey league tracks the plus-minus records for each player. A plus-minus record is the difference in even strength goals for and against the team when a player is on the ice. The following table lists the plus-minus values for several hockey players.

Player	A. Jones	B. Sutter	E. Simpson	L. Mays	R. Tomas	S. Klatt
Plus-minus	−8	4	9	−3	−4	3

a. Graph the values on the number line.
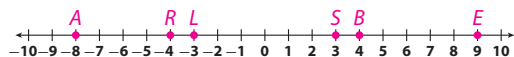
b. Which player has the best plus-minus record? ____ E. Simpson

Astronomy The table lists the average surface temperature of some planets. Write an inequality to compare the temperatures of each pair of planets.

Planet	Average Surface Temperature (°C)
Mercury	167
Uranus	−197
Neptune	−200
Earth	15
Mars	−65
Jupiter	−110

16. Uranus and Jupiter ____ $-197 < -110$

17. Mercury and Mars ____ $167 > -65$

18. Arrange the planets in order of average surface temperature from greatest to least. Mercury, Earth, Mars, Jupiter, Uranus, Neptune

19. **Represent Real-World Problems** For a stock market project, five students each invested pretend money in one stock. They tracked gains and losses in the value of that stock for one week. In the following table, a gain is represented by a positive number and a loss is represented by a negative number.

Students	Andre	Bria	Carla	Daniel	Ethan
Gains and Losses ($)	7	−2	−5	2	4

Graph the students' results on the number line. Then list them in order from least to greatest.

a. Graph the values on the number line.
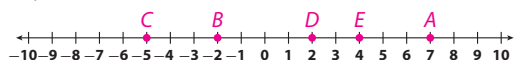
b. The results listed from least to greatest are ____ $-5, -2, 2, 4, 7$.

Geography The table lists the lowest elevation for several countries. A negative number means the elevation is below sea level, and a positive number means the elevation is above sea level. Compare the lowest elevation for each pair of countries. Write < or >.

Country	Lowest Elevation (feet)
Argentina	−344
Australia	−49
Czech Republic	377
Hungary	249
United States	−281

20. Argentina and the United States $-344 < -281$

21. Czech Republic and Hungary $377 > 249$

22. Hungary and Argentina $249 > -344$

23. Which country in the table has the lowest elevation? ____ Argentina

24. **Analyze Relationships** There are three numbers a, b, and c, where $a > b$ and $b > c$. Describe the positions of the numbers on a number line. The first number, a, will be the farthest to the right on the number line. The third number, c, will be farthest to the left on the number line. The second number, b, will be between a and c on the number line.

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

25. **Critique Reasoning** At 9 A.M. the outside temperature was −3°F. By noon, the temperature was −12°F. Jorge said that it was getting warmer outside. Is he correct? Explain. No; $-12°F < -3°F$, so it was getting colder outside.

26. **Problem Solving** Golf scores represent the number of strokes above or below par. A negative score means that you hit a number below par while a positive score means that you hit a number above par. The winner in golf has the lowest score. During a round of golf, Angela's score was −5 and Lisa's score was −8. Who won the game? Explain. Lisa won the game because she had the lowest score.

27. **Look for a Pattern** Order −3, 5, 16, and −10 from least to greatest. Then order the same numbers from closest to zero to farthest from zero. Describe how your lists are similar. Would this be true if the numbers were −3, 5, −16 and −10? $-10, -3, 5, 16$ and $-3, 5, -10, 16$; both lists end with 16 because 16 is the greatest number and is farthest from zero. This would not be true for the second group of numbers because in that list, the least number, −16, would be farthest from zero.

EXTEND THE MATH   PRE-AP

Activity available online   my.hrw.com

Activity In a game on a number line, the starting line is at zero. Each player makes three consecutive jumps. A forward jump is represented by a positive number, and a backward jump is represented by a negative number.

1st jump: The player makes a jump away from the starting line and lands on a point.

2nd jump: From the point where he or she lands, the player makes a second jump towards the starting line.

3rd jump: Now the player makes a third jump away from the starting line.

Rachel makes three consecutive jumps of 8 feet, −5 feet, and 6 feet.

Andy makes three consecutive jumps of −10 feet, 7 feet, and −4 feet.

Who is closest to the starting line at the end of the round? Explain. You may find it helpful to use a counter and a number line to track each player's jumps.

Rachel jumps forward 8 ft, then back 5 ft, and then forward 6 ft. She is now 9 ft in front of the starting line. Andy jumps back 10 ft, then forward 7 ft, and then back 4 ft. He is now 7 ft behind the starting line. Since −7 is closer to zero than 9, Andy is closest to the starting line at the end of the round.

ADDITIONAL EXAMPLE 1
A deep-sea diver dived off a boat to a depth of −45 feet. What is the absolute value that expresses the distance the diver went? The absolute value of −45 is 45.

Interactive Whiteboard
Interactive example available online

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Animated Math
Absolute Values and Opposites

Students explore integers, their opposites, and their absolute values with a dynamic number line.

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Engage

ESSENTIAL QUESTION
How do you find and use absolute value? Count the distance from zero to a number on a number line. Absolute value is always nonnegative and is useful for representing distance or an amount of change.

Motivate the Lesson
Ask: Have you ever borrowed money from a friend? How can you mathematically describe owing money? Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1
Connect to Daily Life
Point out to students that they can use absolute value to describe or compare real-life distances such as how far they ride a bike, dive under water, or ascend in a Ferris wheel.

Explain

EXAMPLE 1
Focus on Communication
Discuss with students why an absolute value of $25 may be used to describe a −$25 change to the balance of a gift card.

Engage with the Whiteboard
Have students take turns graphing a number on a number line and showing how to use the number line to find the absolute value of the number.

Questioning Strategies Mathematical Processes
- How would you define a balance on a gift card? It is the amount of money that is available to the cardholder.
- How would you explain what a balance of $0.00 on a gift card means? It means that the card has no monetary value.
- How can you use absolute value to show the amount Jake has left on his gift card? Find the absolute value of each item Jake bought. Add the two values to find the absolute value of his purchases. Subtract that number from the balance on his gift card.

1.3 Absolute Value
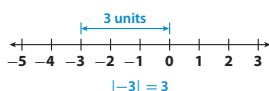
? ESSENTIAL QUESTION

How do you find and use absolute value?


EXPLORE ACTIVITY 1 TEKS 6.2.B

Finding Absolute Value

The **absolute value** of a number is the number's distance from 0 on a number line. For example, the absolute value of -3 is 3 because -3 is 3 units from 0. The absolute value of -3 is written $|-3|$.

3 units

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3$

$|-3| = 3$

Because absolute value represents a distance, it is always nonnegative.

Graph the following numbers on the number line. Then use your number line to find each absolute value.

$-7 \quad 5 \quad 7 \quad -2 \quad 4 \quad -4$

$-10\ -9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10$
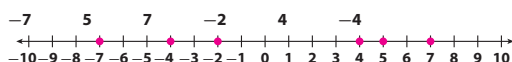
A $|-7| = \underline{7}$ **B** $|5| = \underline{5}$ **C** $|7| = \underline{7}$

D $|-2| = \underline{2}$ **E** $|4| = \underline{4}$ **F** $|-4| = \underline{4}$

Reflect

1. **Analyze Relationships** Which pairs of numbers have the same absolute value? How are these numbers related?

   -7 and 7; 4 and -4; they are opposites.

2. **Justify Reasoning** Negative numbers are less than positive numbers. Does this mean that the absolute value of a negative number must be less than the absolute value of a positive number? Explain.

   No; $-7 < 3$ but $|-7| > |3|$; the distance from -7 to 0 is greater than the distance from 3 to 0.

Math On the Spot
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Animated Math
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Absolute Value In A Real-World Situation

In real-world situations, absolute values are often used instead of negative numbers. For example, if you use a $50 gift card to make a $25 purchase, the change in your gift card balance can be represented by $-$25$.

EXAMPLE 1 Real World   TEKS 6.2.B

Jake uses his online music store gift card to buy an album of songs by his favorite band.
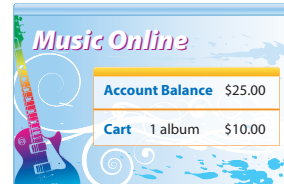
Find the negative number that represents the change in the balance on Jake's card after his purchase. Explain how absolute value would be used to express that number in this situation.

Music Online

Account Balance	$25.00
Cart 1 album	$10.00

STEP 1 Find the negative integer that represents the change in the balance.

$-$10$ *The balance decreased by $10, so use a negative number.*

The balance went down, so the change is a negative number.

Math Talk
Mathematical Processes
Explain why the price Jake paid for the album is represented by a negative number.

Jake pays $10 so there will be a change of $-$10$ in Jake's gift card balance.

STEP 2 Use the number line to find the absolute value of $-$10$.

$-$10 is 10 units from 0 on the number line.

10 units

$-10\ -9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10$
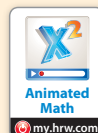
The absolute value of $-$10$ is $10, or $|-10| = 10$.

The balance on Jake's card decreased by $10.

Reflect

3. **Communicate Mathematical Ideas** Explain why the absolute value of a number will never be negative.

   The absolute value of a number is its distance from 0 on the number line. Distance can never be negative.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Processes

This lesson provides an opportunity to address Mathematical Process **TEKS 6.1.A,** which calls for students to "apply mathematics to problems arising in everyday life, society, and the workplace." Example 1 and Explore Activity 2 draw direct connections between absolute value and real-world situations, including the amount owed on a credit card and the amount of money stored on a gift card.

Math Background

You can interpret absolute value as the magnitude of a real number without regard to its sign. It measures the amount of change rather than the direction of change; the farther a number is from 0, the greater its absolute value. This is easy to visualize on a number line. You can also look at it mathematically:

$|n| = n$ if n is ≥ 0

$|n| = -n$ if n is < 0

YOUR TURN

Avoid Common Errors
Make sure that students understand that the absolute value of any negative integer is its *distance* from zero on a number line, which is always expressed as a *nonnegative* number.

Talk About It
Check for Understanding

Ask: What can you say about the distance of numbers −55 and 55 from 0? Because they are opposites, they are both the same distance from 0 and have the same absolute value.

EXPLORE ACTIVITY 2

Connect Vocabulary ELL
Point out to students that when working with money, a loss or a debt can be represented by a negative number. So, in Explore Activity 2, the negative amounts represent money that you spent, a negative change.

Talk About It
Check for Understanding

Ask: How can you tell which person owes the most money? His or her balance will have the greatest absolute value.

Questioning Strategies Mathematical Processes

- If a person has a credit card balance of $50 and has a −$30 change in their balance, how do you find the amount the person owes? Find the absolute value of −$30, which is $30, and add it to $50. The person now owes $80.

- If a person's credit card balance decreases, what happens to the amount the person owes? It decreases.

- When a person makes a payment on their credit card, what happens to the amount of money available on the card (card limit) and to the amount the person owes (card balance)? The amount of money available (card limit) will increase while the amount the person owes (card balance) will decrease.

Elaborate

Talk About It
Summarize the Lesson

Ask: How do you use absolute value to compare two negative numbers, such as fees, or amounts owed on a credit card or other kind of loan? You compare the absolute values of the negative numbers; the negative number with the greater absolute value is the lesser amount, indicating a greater amount owed.

GUIDED PRACTICE

Engage with the Whiteboard
For Exercise 2, have students count the tick marks to show that the distance from 0 to −10 is 10.

Avoid Common Errors
Exercise 2 If students have difficulty understanding how a credit or a fee affects the bill, remind them that a credit is like a payment, it will decrease the balance, while a fee is like a purchase, it will increase the balance.
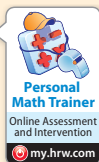
YOUR TURN

4. The temperature at night reached $-13\,°F$. Write an equivalent statement about the temperature using the absolute value of the number.

    The temperature at night reached 13°F below zero.

Find each absolute value.

5. $|-12|$ ____12____
6. $|91|$ ____91____
7. $|-55|$ ____55____
8. $|0|$ ____0____
9. $|88|$ ____88____
10. $|1|$ ____1____

EXPLORE ACTIVITY 2 *Real World* — **TEKS** 6.2.B

Comparing Absolute Values

You can use absolute values to compare negative numbers in real-world situations.

Maria, Susan, George, and Antonio checked their credit card balances on their smartphones. The amounts owed are shown.

You owe: $20	You owe: $25	You owe: $30	You owe: $45
Susan	George	Antonio	Maria

Answer the following questions. When you have finished, you will have enough clues to match each statement with the correct person.

Remember: When someone owes a positive amount of money, this means that he or she has a *negative* balance.

A Maria's credit card balance is less than -30. Does Maria owe more than $30 or less than $30? ____more than $30____

B Susan's credit card balance is greater than -25. Does Susan owe more than $25 or less than $25? ____less than $25____

C George's credit card balance is $5 less than Susan's balance. Does George owe more than Susan or less than Susan? ____more than Susan____

D Antonio owes $15 less than Maria owes. This means that Antonio's balance is ____greater____ than Maria's balance.

E Write each person's name underneath his or her smartphone.

Reflect

11. **Analyze Relationships** Use absolute value to describe the relationship between a negative credit card balance and the amount owed.

    The amount owed is the absolute value of the balance.

Guided Practice

1. **Vocabulary** If a number is ____negative____, then the number is less than its absolute value. *(Explore Activity 1)*

2. If Ryan pays his car insurance for the year in full, he will get a credit of $28. If he chooses to pay a monthly premium, he will pay a $10 late fee for any month that the payment is late. *(Explore Activity 1, Example 1)*

    a. Which of these values could be represented with a negative number? Explain.

    $-$$10; it is a fee, so it represents a change of $-$$10 in the amount of money Ryan has.

    b. Use the number line to find the absolute value of your answer from part a. ____$|-10| = 10$____

    $-10\ -9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10$

3. Leo, Gabrielle, Sinea, and Tomas are playing a video game. Their scores are described in the table below. *(Explore Activity 2)*

Name	Leo	Gabrielle	Sinea
Score	less than -100 points	20 more points than Leo	50 points less than Leo

   a. Leo wants to earn enough points to have a positive score. Does he need to earn more than 100 points or less than 100 points? ____more than 100____

   b. Gabrielle wants to earn enough points to not have a negative score. Does she need to earn more points than Leo or less points than Leo? ____less than Leo____

   c. Sinea wants to earn enough points to have a higher score than Leo. Does she need to earn more than 50 points or less than 50 points? ____more than 50____

? ESSENTIAL QUESTION CHECK-IN

4. When is the absolute value of a number equal to the number?

    when the number is nonnegative

DIFFERENTIATE INSTRUCTION

Home Connection

Students may be unfamiliar with how loans work. Discuss that many people borrow money they need to buy expensive items like cars, furniture, computers, and homes. Discuss that people pay back the money they borrow over a period of time and they pay fees for that privilege. Invite students to cite some examples with which they are familiar. Then have them define absolute value in their own words and then explain how it is used to express the amount of money borrowed.

Critical Thinking

Ask: How does the relationship between a negative number and its absolute value compare with the relationship between a nonnegative number and its absolute value? A nonnegative number is *equal to* its absolute value; a negative number is *less than* its absolute value.

Additional Resources

Differentiated Instruction includes:

• Reading Strategies
• Success for English Learners **ELL**
• Reteach
• Challenge **PRE-AP**

Personal
Math Trainer

Online Assessment
and Intervention

Online homework
assignment available

⏻ my.hrw.com

1. Mia's credit card balance is less than −$85. Does she owe more or less than $85?

2. Leon has a gift card for $100. He spent $65 of it on books. Describe the change in Leon's card balance in two different ways.

3. The record low temperature in Oregon is −54 °F. Use absolute value to express that temperature in degrees below zero.

4. Nick's bank account balance changed by $34 one month and by −$82 the next month. Which amount represents the lesser change?

Lesson Quiz available online

⏻ **my.hrw.com**

Answers

1. She owes more than $85.

2. Use the negative number −$65 to represent the change in the value of Leon's card; use absolute value to say that his balance will be $65 less.

3. 54 degrees below zero

4. $34

Evaluate

GUIDED AND INDEPENDENT PRACTICE

🔲 **TEKS** 6.2.B

Concepts & Skills	Practice
Explore Activity 1 **Finding Absolute Value**	Exercises 1, 2
Example 1 **Absolute Value in a Real-World Situation**	Exercises 2, 5, 8–11
Explore Activity 2 **Comparing Absolute Values**	Exercises 3, 6, 7

| Exercise	Depth of Knowledge (D.O.K.)	🔲 **TEKS** Mathematical Processes
5	**2** Skills/Concepts	**1.A** Everyday life
6	**2** Skills/Concepts	**1.F** Analyze relationships
7	**3** Strategic Thinking **H.O.T.**	**1.G** Explain and justify arguments
8	**2** Skills/Concepts	**1.F** Analyze relationships
9	**2** Skills/Concepts	**1.A** Everyday life
10	**2** Skills/Concepts	**1.A** Everyday life
11	**2** Skills/Concepts	**1.A** Everyday life
12	**3** Strategic Thinking **H.O.T.**	**1.F** Analyze relationships
13	**3** Strategic Thinking **H.O.T.**	**1.F** Analyze relationships
14	**3** Strategic Thinking **H.O.T.**	**1.F** Analyze relationships

Additional Resources

Differentiated Instruction includes:

• Leveled Practice Worksheets

1.3 Independent Practice

TEKS 6.2.B
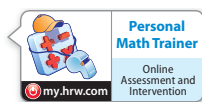
5. Financial Literacy Jacob earned $80 babysitting and deposited the money into his savings account. The next week he spent $85 on video games. Use integers to describe the weekly changes in Jacob's savings account balance.

The first week his balance changed by +$80. The second week his balance changed by −$85.

6. Financial Literacy Sara's savings account balance changed by $34 one week and by −$67 the next week. Which amount represents the greatest change? −$67

7. Analyze Relationships Bertrand collects movie posters. The number of movie posters in his collection changes each month as he buys and sells posters. The table shows how many posters he bought or sold in the given months.

Month	January	February	March	April
Posters	Sold 20	Bought 12	Bought 22	Sold 28

a. Which months have changes that can be represented by positive numbers? Which months have changes that can be represented by negative numbers? Explain.

February and March represent positive numbers because Bertrand bought posters. January and April represent negative numbers because Bertrand sold posters.

b. According to the table, in which month did the size of Bertrand's poster collection change the most? Use absolute value to explain your answer.

April; He sold 28 posters which can be represented by −28. The absolute value of −28 is 28, the greatest of any month.

8. Earth Science Death Valley has an elevation of −282 feet relative to sea level. Explain how to use absolute value to describe the elevation of Death Valley as a positive integer.

The absolute value of −282 is 282 so Death Valley is 282 feet below sea level.

9. Communicate Mathematical Ideas Lisa and Alice are playing a game. Each player either receives or has to pay play money based on the result of their spin. The table lists how much a player receives or pays for various spins.

Red	Pay $5
Blue	Receive $4
Yellow	Pay $1
Green	Receive $3
Orange	Pay $2

a. Express the amounts in the table as positive and negative numbers.

−5, 4, −1, 3, −2

b. Describe the change to Lisa's amount of money when the spinner lands on red.

The spinner landing on red results in a change of −$5 to Lisa's amount of money.

10. Financial Literacy Sam's credit card balance is less than −$36. Does Sam owe more or less than $36? Sam owes more than $36.

11. Financial Literacy Emily spent $55 from her savings on a new dress. Explain how to describe the change in Emily's savings balance in two different ways.

Use a negative integer to say that Emily's balance changed by −$55; Use absolute value to say that Emily's balance is $55 less.

H.O.T. FOCUS ON HIGHER ORDER THINKING

12. Make a Conjecture Can two different numbers have the same absolute value? If yes, give an example. If no, explain why not.

Yes, it is possible. For example, $|-1| = 1$ and $|1| = 1$.

13. Communicate Mathematical Ideas Does $-|-4| = |-(-4)|$? Justify your answer.

No; $-|-4| = -4$, and $|-(-4)| = |4| = 4$.

14. Critique Reasoning Angelique says that finding the absolute value of a number is the same as finding the opposite of the number. For example, $|-5| = 5$. Explain her error.

Angelique's technique only works if the original number is negative. The absolute value of a nonnegative number is equal to the number itself, not its opposite.

Work Area

EXTEND THE MATH   PRE-AP

Activity available online my.hrw.com

**Activity** Read each statement carefully. Write *True* or *False*.

1. $|14| > 14$ False

2. $|44| = |-44|$ True

3. $|-17| = 17$ True

4. $|-22| = -22$ False

5. $-n$ and n have the same absolute value. True

6. $|-33|$ is the opposite of -33. True

7. Rewrite one number in Exercises 1–4 above to make each false statement true and each true statement false.

1. $|14| > -14$ True

2. $-|44| = |-44|$ False

3. $|-17| = -17$ False

4. $|-22| = 22$ True

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI **Response to Intervention**
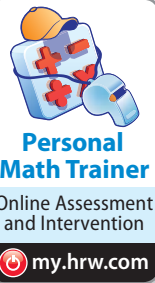
Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Online and Print Resources

Differentiated Instruction	*Differentiated Instruction*
• Reteach worksheets	• Challenge worksheets **PRE-AP**
• Reading Strategies **ELL**	
• Success for English Learners **ELL**	Extend the Math **PRE-AP** Lesson Activities in TE

Additional Resources

Assessment Resources includes*:*
• Leveled Module Quizzes

Ready to Go On?

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com
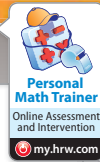
1.1 Identifying Integers and Their Opposites

1. The table shows the elevations in feet of several locations around a coastal town. Graph and label the locations on the number line according to their elevations.

Location	Post Office A	Library B	Town Hall C	Laundromat D	Pet Store E
Elevation (feet)	8	−3	−9	3	1

Write the opposite of each number.

2. −22 ____**22**____   **3.** 0 ____**0**____

1.2 Comparing and Ordering Integers

List the numbers in order from least to greatest.

4. −2, 8, −15, −5, 3, 1   **−15, −5, −2, 1, 3, 8**

Compare. Write < or >.

5. −3 $>$ −15   **6.** 9 $>$ −10

1.3 Absolute Value

Graph each number on the number line. Then use your number line to find each absolute value.
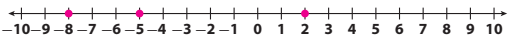
7. $|2|$ ___**2**___   **8.** $|-8|$ ___**8**___   **9.** $|-5|$ ___**5**___

? **ESSENTIAL QUESTION**

10. How can you use absolute value to represent a negative number in a real-world situation?
Sample answer: Sam charged $10 to his credit card. This represents a change of −$10 in his credit card balance.

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Texas Essential Knowledge and Skills

Lesson	Exercises	TEKS
1.1	1–3	**6.2.B**
1.2	4–6	**6.2.C**
1.3	7–10	**6.2.B**

Texas Test Prep

Texas Testing Tip Students can draw a diagram, graph, or picture to help organize information from a test item.

Item 5 If students sketch a number line and plot a point for the temperature of each city, Calgary's point will be the farthest to the left. This means Calgary is the coldest, and therefore the correct answer.

Item 6 If students notice that each answer choice uses the same numbers in a different order, they can sketch a number line and plot the numbers from any of the answer choices. Reading the plotted points from left to right gives the order of the numbers from least to greatest, revealing C as the correct answer.

Avoid Common Errors

Item 2 Students may read the term *opposite* and think that the answer will be negative. Point out that they need to find the opposite of negative 3, which is positive 3.

Item 7 Caution students to read the question carefully so they understand what is being asked. The question asks for the numbers to be ordered from greatest to least rather than from least to greatest.
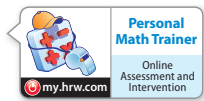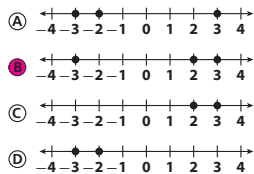
Selected Response

1. Which number line shows 2, 3, and −3?

Ⓐ −4 −3 −2 −1 0 1 2 3 4

Ⓑ −4 −3 −2 −1 0 1 2 3 4

Ⓒ −4 −3 −2 −1 0 1 2 3 4

Ⓓ −4 −3 −2 −1 0 1 2 3 4

2. What is the opposite of −3?

Ⓐ 3   Ⓒ $-\frac{1}{3}$

Ⓑ 0   Ⓓ $\frac{1}{3}$

3. Darrel is currently 20 feet below sea level. Which correctly describes the opposite of Darrel's elevation?

Ⓐ 20 feet below sea level

Ⓑ 20 feet above sea level

Ⓒ 2 feet below sea level

Ⓓ At sea level

4. Which has the same absolute value as −55?

Ⓐ 0   Ⓒ 1

Ⓑ −1   Ⓓ 55

5. In Bangor it is −3 °F, in Fairbanks it is −12 °F, in Fargo it is −8 °F, and in Calgary it is −15 °F. In which city is it the coldest?

Ⓐ Bangor   Ⓒ Fargo

Ⓑ Fairbanks   Ⓓ Calgary

6. Which shows the integers in order from least to greatest?

Ⓐ 20, 6, −2, −13   Ⓒ −13, −2, 6, 20

Ⓑ −2, 6, −13, 20   Ⓓ 20, −13, 6, −2

7. How would you use a number line to put integers in order from greatest to least?

Ⓐ Graph the integers, then read them from left to right.

Ⓑ Graph the integers, then read them from right to left.

Ⓒ Graph the absolute values of the integers, then read them from left to right.

Ⓓ Graph the absolute values of the integers, then read them from right to left.

Gridded Response

8. The table shows the change in several savings accounts over the past month. Which value represents the least change?

Account	Change
A	$25
B	−$45
C	−$302
D	$108

Texas Essential Knowledge and Skills

Items	🏴 Grade 6 TEKS	🏴 Mathematical Process TEKS
1*	6.2.C	6.1.D
2	6.2.B	6.1.F
3	6.2.B	6.1.A
4	6.2.B	6.1.F
5	6.2.C	6.1.A
6	6.2.C	6.1.E
7	6.2.C	6.1.F, 6.1.G
8	6.2.C	6.1.A

* Item integrates mixed review concepts from previous modules or a previous course.