Parent function

\[ f(x) = b^x, \quad b > 0, \quad b \neq 1 \]

Translation

\[ y = b^x + d \]

- \( d > 0 \) shifts up \(|d|\) units
- \( d < 0 \) shifts down \(|d|\) units

\[ y = b^{x-c} \]

- \( c > 0 \) shifts to the right \(|c|\) units
- \( c < 0 \) shifts to the left \(|c|\) units

Stretch, Compression, and Reflection

\[ y = ab^x \]

- \(|a| > 1\) vertical stretch
- \(0 < |a| < 1\) vertical compression (shrink)
- \(a < 0\) reflection across the \(x\)-axis

ESSENTIAL UNDERSTANDING

You can apply the four types of transformations—stretches, compressions, reflections, and translations—to exponential functions.

Concept Summary Exponential Function Family
Analyzing $y = af(x)$ for $f(x) = 2^x$

Graph each function on the same set of axes as the parent function $f(x) = 2^x$.

What is the effect of the transformation on the $y$-intercept?

A $y = 3 \cdot 2^x$

$a = 3$, so each $y$-value of $y = 3 \cdot 2^x$ is 3 times the corresponding $y$-value of $f(x)$. $y = 3 \cdot 2^x$ stretches the graph of $f(x)$ by the factor 3.

The $y$-intercept is now $(0, 3)$ instead of $(0, 1)$.

B $y = -\frac{1}{3} \cdot 2^x$

$a = -\frac{1}{3}$. Since $a < 0$, $y = -\frac{1}{3} \cdot 2^x$ reflects the graph of $f(x)$ across the $x$-axis and compresses it by the factor $\frac{1}{3}$.

The $y$-intercept is now $(0, -\frac{1}{3})$ instead of $(0, 1)$.

Analyzing $y = af(x)$ for $f(x) = 10^x$

Graph each function on the same set of axes as the parent function $f(x) = 10^x$.

What is the effect of the transformation on the range?

A $y = \frac{1}{2} \cdot 10^x$

$a = \frac{1}{2}$, so each $y$-value of $y = \frac{1}{2} \cdot 10^x$ is one-half the corresponding $y$-value of $f(x)$. $y = \frac{1}{2} \cdot 10^x$ compresses the graph of $f(x)$ by the factor $\frac{1}{2}$.

The range remains $y > 0$.

B $y = -2 \cdot 10^x$

$a = -2$. Since $a < 0$, $y = -2 \cdot 10^x$ reflects the graph of $f(x)$ across the $x$-axis and stretches it by the factor 2.

The range is now $y < 0$ instead of $y > 0$. 

How can you use the parent function to graph $y = 3 \cdot 2^x$?

Multiply each $y$-value of the parent function by a value of 3.

How do you know the graphs are reasonable?

The graph of $y = a \cdot 10^x$ should lie entirely above the $x$-axis when $a > 0$ and entirely below the $x$-axis when $a < 0$. 

Lesson 7-2  Transformations of Exponential Functions
**Problem 3**

Analyzing $y = f(x) + d$ for $f(x) = 2^x$

Graph each function on the same set of axes as the parent function $f(x) = 2^x$.

What is the effect of the transformation on the $y$-intercept?

A $y = 2^x + 3$

- $d = 3$. Since $d > 0$, each $y$-value of $y = 2^x + 3$ is 3 greater than the corresponding $y$-value of $f(x)$. So, $y = 2^x + 3$ shifts the graph of $f(x)$ up by 3 units.

The $y$-intercept is now (0, 4) instead of (0, 1).

B $y = 2^x - 2$

- $d = -2$. Since $d < 0$, $y = 2^x - 2$ shifts the graph of $f(x)$ down by 2 units.

The $y$-intercept is now (0, -1) instead of (0, 1).

**Problem 4**

Analyzing $y = f(x) + d$ for $f(x) = 10^x$

Graph each function on the same set of axes as the parent function $f(x) = 10^x$.

What is the effect of the transformation on the asymptote?

A $y = 10^x + 20$

- Since $d = 20$, each $y$-value of $y = 10^x + 20$ is 20 greater than the corresponding $y$-value of $f(x)$. Because $d > 0$, $y = 10^x + 20$ shifts the graph of $f(x)$ up by 20 units.

The horizontal asymptote is now $y = 20$ instead of $y = 0$.

B $y = 10^x - 40$

- Since $d = -40$ and $d < 0$, $y = 10^x - 40$ shifts the graph of $f(x)$ down by 40 units.

The horizontal asymptote is now $y = -40$ instead of $y = 0$.

**Think**

How do you know the graphs are the correct distance apart?

For vertical translations, measure the distance between the graphs along a vertical line, such as the $y$-axis.

How can you use estimation to help make the graph?

For negative values of $x$, $10^x$ is close to 0. Therefore, to the left of the $y$-axis, the graph of $y = 10^x + 20$ should approach the line $y = 20$. 

[Diagram of graphs showing the transformations]
Problem 5

Analyzing \( y = f(x - c) \) for \( f(x) = 2^x \)

Graph each function on the same set of axes as the parent function \( f(x) = 2^x \).

What is the effect of the transformation on the \( y \)-intercept?

A \( y = 2^{(x-4)} \)

Since \( c = 4 \) and \( c > 0 \), \( y = 2^{(x-4)} \) shifts the graph of \( f(x) \) to the right by 4 units.

B \( y = 2^{(x+2)} \)

Since \( c = -2 \) and \( c < 0 \), \( y = 2^{(x+2)} \) shifts the graph of \( f(x) \) to the left by \( | -2 | \) or 2 units.

The \( y \)-intercept is now \((0, \frac{1}{16})\) instead of \((0, 1)\).

The \( y \)-intercept is now \((0, 4)\) instead of \((0, 1)\).

Problem 6

Analyzing \( y = f(x - c) \) for \( f(x) = 10^x \)

Graph each function on the same set of axes as the parent function \( f(x) = 10^x \).

What is the effect of the transformation on the domain?

A \( y = 10^{(x-6)} \)

Since \( c = 6 \) and \( c > 0 \), \( y = 10^{(x-6)} \) shifts the graph of \( f(x) \) to the right by 6 units.

B \( y = 10^{(x+5)} \)

Since \( c = -5 \) and \( c < 0 \), \( y = 10^{(x+5)} \) shifts the graph of \( f(x) \) to the left by \( | -5 | \) or 5 units.

The domain of the function remains \( \{x \mid x \text{ is a real number}\} \).

The domain of the function remains \( \{x \mid x \text{ is a real number}\} \).
Problem 7

Using an Exponential Model

**Physics** The best temperature to brew coffee is between 195°F and 205°F. Coffee is cool enough to drink at 185°F. The table shows temperature readings from a sample cup of coffee. How long does it take for a cup of coffee to be cool enough to drink? Use an exponential model.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>203</td>
</tr>
<tr>
<td>5</td>
<td>177</td>
</tr>
<tr>
<td>10</td>
<td>153</td>
</tr>
<tr>
<td>15</td>
<td>137</td>
</tr>
<tr>
<td>20</td>
<td>121</td>
</tr>
<tr>
<td>25</td>
<td>111</td>
</tr>
<tr>
<td>30</td>
<td>104</td>
</tr>
</tbody>
</table>

**Know**
- Set of values
- Best serving temperature

**Need**
- Time it takes for a cup of coffee to become cool enough to drink

**Plan**
- Use an exponential model to find the time it takes for coffee to reach 185°F.

**Step 1**
Plot the data to determine if an exponential model is realistic.

**Step 2**
The graphing calculator exponential model assumes the asymptote is $y = 0$. Since room temperature is about 68°F, subtract 68 from each temperature value. Calculate the third list by letting $L3 = L2 - 68$.

**Step 3**
Use the ExpReg L1, L3 function on the transformed data to find an exponential model.

**ExpReg**

- $y = a \cdot b^x$
- $a = 134.5169825$
- $b = 0.956011669$
- $r^2 = 0.9981659939$
- $r = -0.9990825761$

**Step 4**
Translate $y = 134.5(0.956)^x$ vertically by 68 units to model the original data. Use the model $y = 134.5 \cdot 0.956^x + 68$ to find how long it takes the coffee to cool to 185°F.

The coffee takes about 3.1 min to cool to 185°F.
1. Select Tools to Solve Problems (1)(C) A bread recipe says to bake the bread until the center is 180°F, then let the bread cool to 120°F. The table shows temperature readings for the bread.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (°F)</td>
<td>180</td>
<td>126</td>
<td>94</td>
<td>80</td>
<td>73</td>
</tr>
</tbody>
</table>

a. Given a room temperature of 70°F, what is an exponential model for this data set? Use technology.

b. How long does it take the bread to cool to the desired temperature?

Graph each function on the same set of axes as the parent function. What is the effect of the transformation on the domain, range, x-intercept, y-intercept, and asymptote?

2. \( y = \frac{1}{2} \cdot 10^x \)
3. \( y = 2^{(x-1)} \)
4. \( y = -\frac{1}{4} \cdot 2^x \)
5. \( y = 10^x + 30 \)
6. \( y = 10^{(x-2)} \)
7. \( y = 4 \cdot 2^x \)
8. \( y = 2^x - 3 \)
9. \( y = 10^x - 20 \)
10. \( y = -\frac{1}{2} \cdot 10^x \)
11. \( y = 2^x + 4 \)
12. \( y = 10^{(x+1)} \)
13. \( y = 2^{(x+5)} \)

14. Water boils at 212°F. As part of an experiment, you boil water and let it cool, taking temperature readings every 3 minutes. The table shows the data.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (°F)</td>
<td>212</td>
<td>193</td>
<td>180</td>
<td>169</td>
<td>160</td>
</tr>
</tbody>
</table>

a. Given a room temperature of 72°F, what is an exponential model for this data set?

b. What is the y-intercept of the model? What does it represent?

c. What is the approximate temperature of the water after 15 minutes?

d. How long does it take the water to cool to a temperature of 140°F?

15. Analyze Mathematical Relationships (1)(F) Without actually graphing, describe how the graph of \( y = -2^{(x+17)} - 4.3 \) is related to the graph of \( y = 2^x \).

Determine whether each statement is true or false and explain your answer.

16. The function \( y = 10^{(x-2)} \) has the same domain and range as the parent function \( f(x) = 10^x \).

17. The function \( y = 2^x + 1 \) has the same domain and range as the parent function \( f(x) = 2^x \).

18. The function \( y = 2^{(x-3)} \) has the asymptote \( y = -3 \).

19. The range of the function \( y = -10^x \) is \((-\infty, 0)\).

20. The y-intercept of the function \( y = 2^x + 6 \) is \((0, 6)\).
21. **Explain Mathematical Ideas (1)(G)** A student says that the graph of \( y = 2^{(x+2)} + 3 \) is a shift of the parent function 2 units to the right and 3 units up. Is the student correct? Explain.

Match each function with the correct graph.

22. \( y = 2^{(x-1)} + 1 \)  
23. \( y = 2^{(x+1)} - 1 \)  
24. \( y = 2^{(x-1)} - 1 \)

25. A student graphed a function of the form \( y = 2^{(x-c)} + d \). She gave the following clues about the graph.
   - The \( y \)-intercept is \((0, 4)\).
   - The asymptote is the \( x \)-axis.
   - The graph passes through the point \((-1, 2)\).

Write a rule for the function and then graph the function.

26. Which function has a graph that lies entirely in the third and fourth quadrants?
   - A. \( y = 10^x - 1 \)
   - B. \( y = -10^x + 1 \)
   - C. \( y = -10^{(x-1)} - 1 \)
   - D. \( y = 10^{(x-1)} - 1 \)

27. A teacher graphed the parent function \( f(x) = 2^x \). Then she shifted the graph of the parent function 5 units left and 5 units up. Which function did she graph?
   - F. \( y = 2^{(x+5)} + 5 \)
   - G. \( y = 2^{(x-5)} + 5 \)
   - H. \( y = 2^{(x+5)} - 5 \)
   - J. \( y = 2^{(x-5)} - 5 \)

28. Explain how the graph of the function \( y = -0.2 \cdot 2^x \) is similar to and different from the graph of the function \( y = 0.2 \cdot 2^x \).