12-1 Tangent Lines

TEKS FOCUS

TEKS (12)(A) Apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems.

TEKS (1)(F) **Analyze** mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(G), (6)(A), (9)(B)

VOCABULARY

- **Point of tangency** – the point where a circle and a tangent intersect
- **Tangent to a circle** – a line in the plane of the circle that intersects the circle in exactly one point

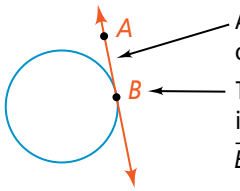
- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature

ESSENTIAL UNDERSTANDING

A radius of a circle and the tangent that intersects the endpoint of the radius on the circle have a special relationship.

take note

Key Concept Tangent Lines



A **tangent to a circle** is a line in the plane of the circle that intersects the circle in exactly one point.

The point where a circle and a tangent intersect is the **point of tangency.**

$\overrightarrow{BA}$ is a tangent ray, and $\overline{BA}$ is a tangent segment.

take note

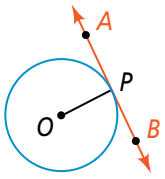
Theorem 12-1

Theorem

If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.

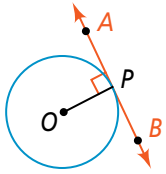
If . . .

$\overleftrightarrow{AB}$ is tangent to $\odot O$ at P



Then . . .

$\overleftrightarrow{AB} \perp \overline{OP}$



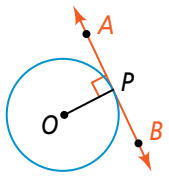
For a proof of Theorem 12-1, see the Reference section on page 683.

take note

Theorem 12-2

Theorem
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

If . . .
$\overleftrightarrow{AB} \perp \overline{OP}$ at P



Then . . .
$\overleftrightarrow{AB}$ is tangent to $\odot O$

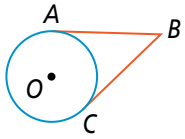
You will prove Theorem 12-2 in Exercise 19.

take note

Theorem 12-3

Theorem
If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

If . . .
$\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$



Then . . .
$\overline{BA} \cong \overline{BC}$

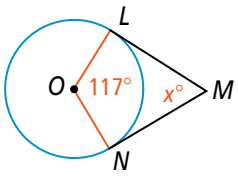
You will prove Theorem 12-3 in Exercise 12.

Problem 1

TEKS Process Standard (1)(F)

Finding Angle Measures

Multiple Choice $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot O$. **What is the value of x?**



Ⓐ 58 Ⓑ 63 Ⓒ 90 Ⓓ 117

Think

What kind of angle is formed by a radius and a tangent?
The angle formed is a right angle, so the measure is 90.

Since $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot O$, $\angle L$ and $\angle N$ are right angles (Theorem 12-1). $LMNO$ is a quadrilateral. So the sum of the angle measures is 360.

$$m\angle L + m\angle M + m\angle N + m\angle O = 360$$
$$90 + m\angle M + 90 + 117 = 360 \quad \text{Substitute.}$$
$$297 + m\angle M = 360 \quad \text{Simplify.}$$
$$m\angle M = 63 \quad \text{Solve.}$$

The correct answer is B.

Problem 2

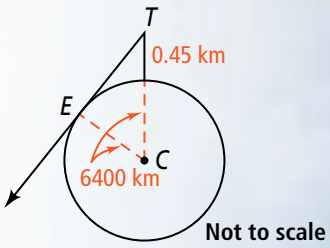
Finding Distance STEM

Earth Science **The CN Tower in Toronto, Canada, has an observation deck 447 m above ground level. About how far is it from the observation deck to the horizon? Earth's radius is about 6400 km.**

Plan

How does knowing Earth's radius help?
The radius forms a right angle with a tangent line from the observation deck to the horizon. So you can use two radii, the tower's height, and the tangent to form a right triangle.

Step 1 Make a sketch. The length 447 m is about 0.45 km.





Step 2 Use the Pythagorean Theorem.

$$CT^2 = TE^2 + CE^2$$

$(6400 + 0.45)^2 = TE^2 + 6400^2$ Substitute.

$(6400.45)^2 = TE^2 + 6400^2$ Simplify.

$40{,}965{,}760.2025 = TE^2 + 40{,}960{,}000$ Use a calculator.

$5760.2025 = TE^2$ Subtract 40,960,000 from each side.

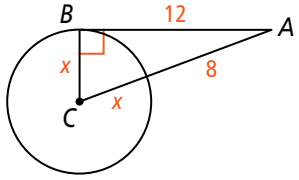
$76 \approx TE$ Take the positive square root of each side.

The distance from the CN Tower to the horizon is about 76 km.

Problem 3

Finding a Radius

What is the radius of $\odot C$?



Think

Why does the value *x* appear on each side of the equation?
The length of $\overline{AC}$, the hypotenuse, is the radius plus 8, which is on the left side of the equation. On the right side of the equation, the radius is one side of the triangle.

$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$(x + 8)^2 = 12^2 + x^2$ Substitute.

$x^2 + 16x + 64 = 144 + x^2$ Simplify.

$16x = 80$ Subtract x^2 and 64 from each side.

$x = 5$ Divide each side by 16.

The radius is 5.

Problem 4

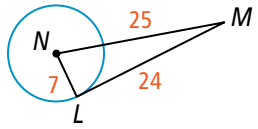
TEKS Process Standard (1)(G)

Identifying a Tangent

Is $\overline{ML}$ tangent to $\odot N$ at L? Explain.

Think

What information does the diagram give you?
- LMN is a triangle.
- $NM = 25$, $LM = 24$, $NL = 7$
- $\overline{NL}$ is a radius.



Know

The lengths of the sides of $\triangle LMN$

Need

To determine whether $\overline{ML}$ is tangent to $\odot N$

Plan

$\overline{ML}$ is a tangent if $\overline{ML} \perp \overline{NL}$. Use the Converse of the Pythagorean Theorem to determine whether $\triangle LMN$ is a right triangle.

$$NL^2 + ML^2 \stackrel{?}{=} NM^2$$

$$7^2 + 24^2 \stackrel{?}{=} 25^2 \qquad \text{Substitute.}$$

$$625 = 625 \qquad \text{Simplify.}$$

By the Converse of the Pythagorean Theorem, $\triangle LMN$ is a right triangle with $\overline{ML} \perp \overline{NL}$. So $\overline{ML}$ is tangent to $\odot N$ at L because it is perpendicular to the radius at the point of tangency (Theorem 12-2).

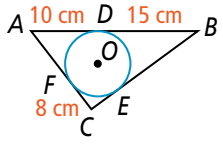
Problem 5

Circles Inscribed in Polygons

$\odot O$ is inscribed in $\triangle ABC$. What is the perimeter of $\triangle ABC$?

Plan

How can you find the length of $\overline{BC}$?
Find the segments congruent to $\overline{BE}$ and $\overline{EC}$. Then use segment addition.



$AD = AF = 10$ cm
$BD = BE = 15$ cm
$CF = CE = 8$ cm

Thm 12-3: Two segments tangent to a circle from a point outside the circle are congruent, so they have the same length.

$P = AB + BC + CA$ Definition of perimeter

$= AD + DB + BE + EC + CF + FA$ Segment Addition Postulate

$= 10 + 15 + 15 + 8 + 8 + 10$ Substitute.

$= 66$

The perimeter is 66 cm.

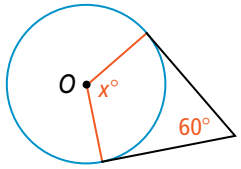
PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to PearsonTEXAS.com.

Lines that appear to be tangent are tangent. *O* is the center of each circle. What is the value of *x*?

1.



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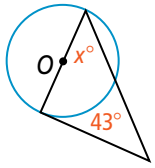

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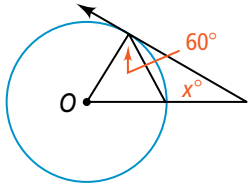
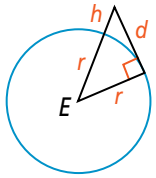


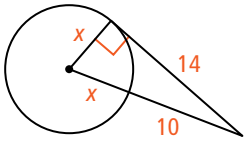
STEM **Apply Mathematics (1)(A)** **The circle at the right represents Earth. The radius of Earth is about 6400 km. Find the distance *d* to the horizon that a person can see on a clear day from each of the following heights *h* above Earth. Round your answer to the nearest tenth of a kilometer.**



4. 5 km **5.** 1 km **6.** 2500 m

Analyze Mathematical Relationships (1)(F) **In each circle, what is the value of *x*, to the nearest tenth?**

7.



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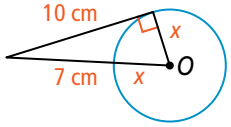

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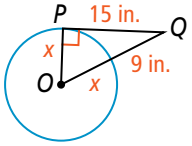
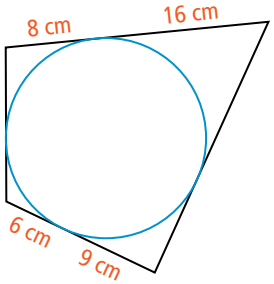
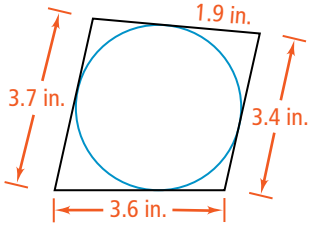


Each polygon circumscribes a circle. What is the perimeter of each polygon?

10.



11.

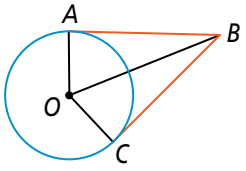


12. Write a paragraph proof to prove Theorem 12-3.

Proof

Given: $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$ at A and C, respectively.

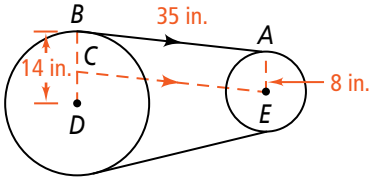
Prove: $\overline{BA} \cong \overline{BC}$



13. a. A belt fits snugly around the two circular pulleys. $\overline{CE}$ is an auxiliary line from E to $\overline{BD}$. $\overline{CE} \parallel \overline{BA}$. What type of quadrilateral is $ABCE$? Explain.

b. What is the length of $\overline{CE}$?

c. What is the distance between the centers of the pulleys to the nearest tenth?



14. Explain Mathematical Ideas (1)(G) A nickel, a dime, and a quarter are touching as shown. Tangents are drawn from point A to both sides of each coin. What can you conclude about the four tangent segments? Explain.

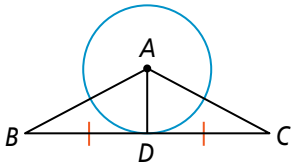


15. Analyze Mathematical Relationships (1)(F) Leonardo da Vinci wrote, "When each of two squares touches the same circle at four points, one is double the other." Explain why the statement is true.

16. Two circles that have one point in common are *tangent circles*. Given any triangle, explain how to draw three circles that are centered at each vertex of the triangle and tangent to each other.

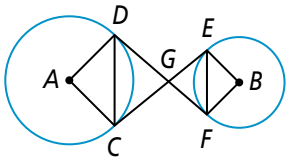
17. Proof **Given:** $\overline{BC}$ is tangent to $\odot A$ at D.
$\overline{DB} \cong \overline{DC}$

Prove: $\overline{AB} \cong \overline{AC}$



18. Proof **Given:** $\odot A$ and $\odot B$ with common tangents $\overline{DF}$ and $\overline{CE}$

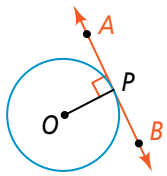
Prove: $\triangle GDC \sim \triangle GFE$



19. Proof Write an indirect proof of Theorem 12-2.

Given: $\overline{AB} \perp \overline{OP}$ at P.

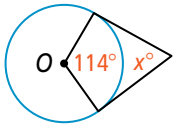
Prove: $\overline{AB}$ is tangent to $\odot O$.



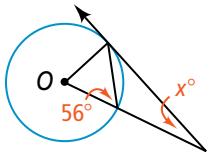
TEXAS Test Practice

Lines in $\odot O$ that appear to be tangent are tangent. What is the value of x?

20.



21.



22. The perimeter of an equilateral triangle is 90 in. What is its area to the nearest square inch?