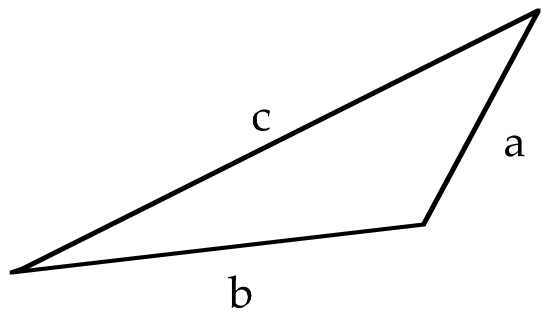
Triangular Frameworks

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meter*s, 3 *meter*s etc and he always keeps one rod of each length in stock.



This diagram shows one of Joe's triangular frameworks.

a, b, c are all integers and $c > b > a$.

That is, c is the longest side, a is the shortest side and a, b, c are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

3. Write down any generalizations you can make.

Triangular Frameworks Rubric		
	Points	Section points
1. Finds examples that match the given general statement, May draw diagrams. For example, when c = 7, b = 6, a = 5.	1	
Searches for patterns and makes statements such as: When c = 7 there are **six** possibilities.	2	3
2. Considers different values of c.	1	
Shows that as c increases the number of triangles increases.	1	
Makes generalizations based on evidence.	1	
The smallest value of c is 4	1	4
3. Searches for patterns.	1	
Uses algebra Notes that when n is even/odd the number of possible triangles is $\frac{(c-2)^2}{4}$ or $\frac{(c-1)(c-3)}{4}$.	2 x 1	3
Total Points		**10**

Triangular Frameworks

T1

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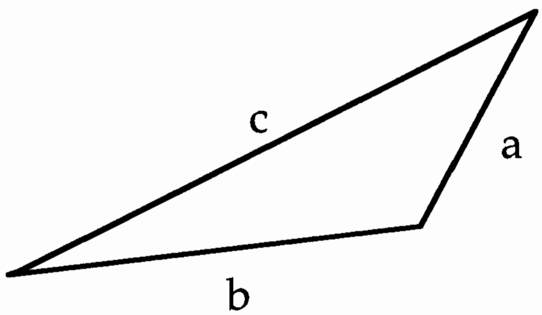

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1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different frameworks

$$\left.\begin{array}{l}7, 6, 5\\7, 6, 4\\7, 6, 3\\7, 6, 2\end{array}\right] 4 \qquad \left.\begin{array}{l}7, 5, 4\\7, 5, 3\end{array}\right] 2$$

2 sides > other side
so only these

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

$c=6$ 6, 5, 4 ; 6, 5, 3 ; 6, 5, 2 — 3 ; 6, 4, 3 — 1 ; $\boxed{4}$

$c=5$ 5, 4, 3 ; 5, 4, 2 — 2 ; ~~5, 3,~~ ; $\boxed{2}$

$c=4$ 4, 3, 2 — 1 ; $\boxed{1}$ c can not < 2

$c=8$ 8, 7, 6 ; 8, 7, 5 ; 8, 7, 4 ; 8, 7, 3 ; 8, 7, 2 — 5 ; 8, 6, 5 ; 8, 6, 4 ; 8, 6, 3 — 3 ; 8, 5, 4 — 1 ; $\boxed{9}$

$c=9$ 9, 8, 7 ; 9, 8, 6 ; 9, 8, 5 ; 9, 8, 4 ; 9, 8, 3 ; 9, 8, 2 — 6 ; 9, 7, 6 ; 9, 7, 5 ; 9, 7, 4 ; 9, 7, 3 — 4 ; 9, 6, 5 ; 9, 6, 4 — 2 ; ~~9, 6, 4~~ ; ~~9, 5~~ ; $\boxed{12}$

can not use a 1 meter rod.

c	n	
4	1	+1
5	2	+2
6	4	+3
7	6	+3
8	9	+3
9	12	

1 → 2 (+3) → 3+1 (+4) → 4+2 (+5) → 5+3+1 (+6) → 6+4+2

3. Write down any generalizations you can make.

$\frac{(c-3)(c-1)}{4}$ if c is odd $\frac{(c-2)^2}{4}$ if c is even

I can't find one rule.

Triangular Frameworks **T2**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

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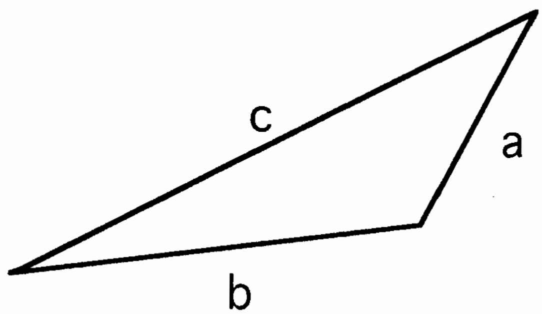

This diagram shows one of Joe's triangular frameworks.

a, b, c are all integers and $c > b > a$.

That is, c is the longest side, a is the shortest side and a, b, c are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 ways

c	b	a
7	6	5 ✓
7	6	4 ✓
7	6	3 ✓
7	6	2 ✓
~~7~~	~~6~~	~~1~~

c	b	a
7	5	4 ✓
7	5	3 ✓
~~7~~	~~5~~	~~2~~
~~7~~	~~5~~	~~1~~

c	b	a
~~7~~	~~4~~	~~3~~
~~7~~	~~4~~	~~2~~
~~7~~	~~4~~	~~1~~

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

$c=8$: 9 ways $\qquad$ $c=4$: 1 ways

$8-2-1=5$ $\qquad$ $5+3+1=9$ $\qquad$ $4-2-1=1$

$8-3-2=3$ $\qquad$ $c=3$

$8-4-3=1$ $\qquad$ bigger c is more combinations.

smallest way = 1 for c = 4

c	b	a	c	b	a	c	b	a
8	7	6	8	6	5	8	5	4
8	7	5	8	6	4	~~8~~	~~5~~	~~3~~
8	7	4	8	6	3	~~8~~	~~5~~	~~2~~
8	7	3	~~8~~	~~6~~	~~2~~	~~8~~	~~5~~	~~1~~
8	7	2	~~8~~	~~6~~	~~1~~			
~~8~~	~~7~~	~~1~~						

c b a
4 3 2

c b a
3 2 1 impossible

3. Write down any generalizations you can make.

total ways $= n \cdot c - n(n+2)$

$c > n+1+n$

① $c-2-1$
② $c-3-2$
③ $c-4-3$
ⓝ $c-(n+1)-n$

Total Ways
$$n \cdot c - \frac{n(n+1+2)}{2} - \frac{(n+1)n}{2}$$
$$= n \cdot c - n(n+2)$$

$c > n+1+n$ $\qquad$ $c > 2n+1$

ex: $c=7$ $\qquad$ $7 > 2n+1$

$n<3$ $\qquad$ $n=2$

Total Ways $= 2 \cdot 7 - 2(4) = 14-8 = 6$

Triangular Frameworks **T3**

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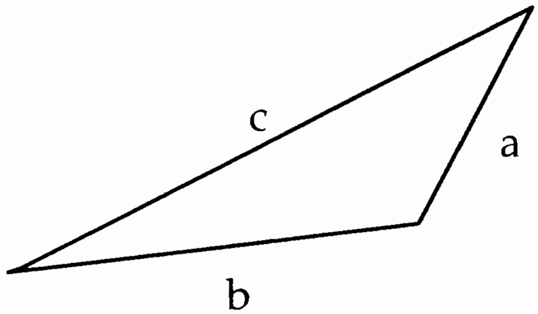

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a, b, c are all integers and $c > b > a$.

That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

c>b>a   b+a MUST be bigger than c

7 7 6 7 5
674
673
672
5 4
5 3
6 ways

T3

Triangular Frameworks (continued)

2. Investigate this situation for other values of c.

$c > 4 > b > a$
3 2

$c > 5$
4 3
4 2

$c > 6$
5 4
5 3
5 2
4 3

$c > 7$

$c > 8$
7 6
7 5
7 4
7 3
7 2
6 5
6 4
6 3
5 4

$c > 9$
8 7
8 6
8 5
8 4
8 3
8 2
7 6
7 5
7 4
7 3
6 5
6 4

$c > 10$
9 8
9 7
9 6
9 5
9 4
9 3
9 2
8 7
8 6
8 5
8 4
8 3
7 6
7 5
7 4
6 5

c	ways	
4	1	
		> 1
5	2	
		> 2
6	4	
		> 2
7	6	
		> 3
8	9	
		> 3
9	12	
		> 4
10	16	

The larger c is, the more triangles can be made

Smallest c is 4 with one triangle

c = 3 is impossible as c > 2 > 1 but 2 + 1 = 3 so no triangle

3. Write down any generalizations you can make.

even numbers go up in square numbers

c	ways	
4	1	1^2
6	4	2^2
8	9	3^2
10	16	4^2

$\frac{(c-2)^2}{4}$ = ways

$10 - 2 = 8$

$8^2 = \frac{64}{4} = 16$ ✓

odds

c	ways
5	2
7	6
9	12

$7 - 2 = 5^2 = \frac{25}{4} = 6+$

$7 - 1 = 6^2 = \frac{36}{4} = 9$

$7 - 3 = 4^2 = \frac{16}{4} = 4$

$\frac{(7-1)(7-3)}{4} = \frac{6 \cdot 4}{4} = 6$ ✓

$\frac{(c-1)(c-3)}{4}$ = ways

Triangular Frameworks

T4

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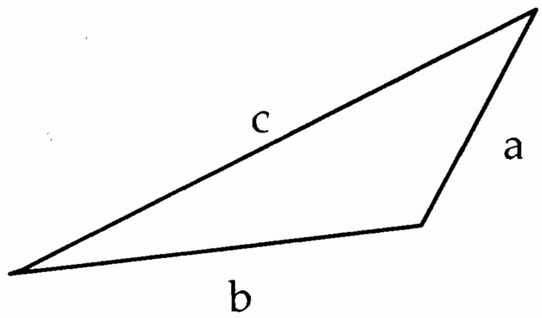

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1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

$a+b>c$ $\quad a+b>7$ $\quad a < b$ $\quad c$

$5 < 6$

$4 < 6$

$3 < 6$

$2 < 6$ $\quad$ 6 ways

$4 < 5$

$3 < 5$

~~2 4~~

T4

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

$c = 8$

a	b	c	a	b	c	c
2	7	8+	2	8	9+	2 3 4+
3	7		3	8		1 way
4	7		4	8		
5	7		5	8		
6	7		6	8		
3	6		7	8		3 4 5+
4	6		6	7		2 4
5	6		5	7		
4	5		4	7		
			3	7		
			4	6		
			5	6		

5 3 1

6 4 2

9 ways

12 ways

As c increases so does the # triangles

c = 4 is smallest # of triangles made

when c = x b = x − 1, a = x − 2 are the largest lengths each can be

3. Write down any generalizations you can make.

when c is even # of ways is odd.

when c is odd # of ways is even

c = 7 6 ways 3 lots of 6 + 2 lots of 5

c = 8 9 ways 5 lots of 7 + 3 lots of 6 + 1 lot of 5

c = 9 12 ways 6 lots of 8 + 4 lots of 7 + 2 lots of 6

c = 4 1 way 1 lot of 3

$\frac{(c-2)^2}{4}$ = even ways.

$8-2 = 6$ $6^2 = 36 \div 4 = 9$

$4-2 = 2$ $2^2 = 4 \div 4 = 1$

Triangular Frameworks **T5**

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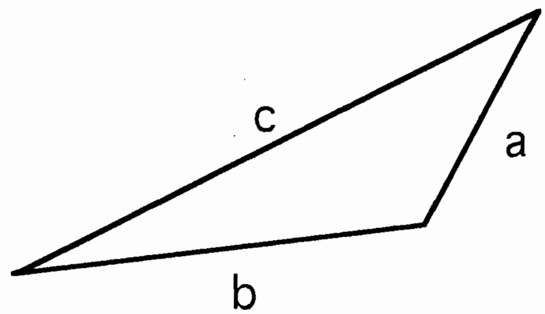

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1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different variations can be made

$7 > 6 > 5$ $7 > 5 > 4$
$7 > 6 > 4$ $7 > 5 > 3$
$7 > 6 > 3$ 2
$7 > 6 > 2$
4

6 ways

Triangular Frameworks (continued)

2. Investigate this situation for other values of c.

8>7>6	8>6>5	6>5>4	5>4>3
8>7>5	8>6>4	6>5>3	5>4>2
8>7>4	~~8>6>3~~	6>5>2	2 ways
8>7>3	8>5>4	6>4>3	
8>7>2	4	4 ways	2 ways
5			4>2>1
9 ways			

9>8>7   9>7>6
9>8>6   9>7>5
9>8>5   9>7>4
9>8>4   9>7>3
9>8>3   9>6>5
9>8>2   9>6>4

12

−3		4	1	
−3		5	2	(+1)
−2		6	4	(+2)
−1		7	6	(+2)
−	1	8	9	(+3)
+2	3	9	12	
+3	6	10	16	
+4	10	11	21	
		12	27	

3. Write down any generalizations you can make.

$c > b > a$ $c-1$ could be   c cannot $=1$

$(a+b) > c$ $(c-2)+(c-b) > c$ $\frac{(c-3)(c-1)}{4}$ odds

$\frac{(4-2)^2}{4} = \frac{4}{4} = 1$ ✓ $\frac{(c-2)^2}{4}$ = w even only ✓

$5-2 = \frac{9}{4}$

$6-2 = \frac{16}{4} = 1$ ✓ $(5-3)(5-1) = \frac{8}{4} = 2$ ✓
2 . 4

$8-2 = \frac{36}{4} = 9$ ✓ $(9-1)(9-3) = \frac{48}{4} = 12$ ✓
8 . 6