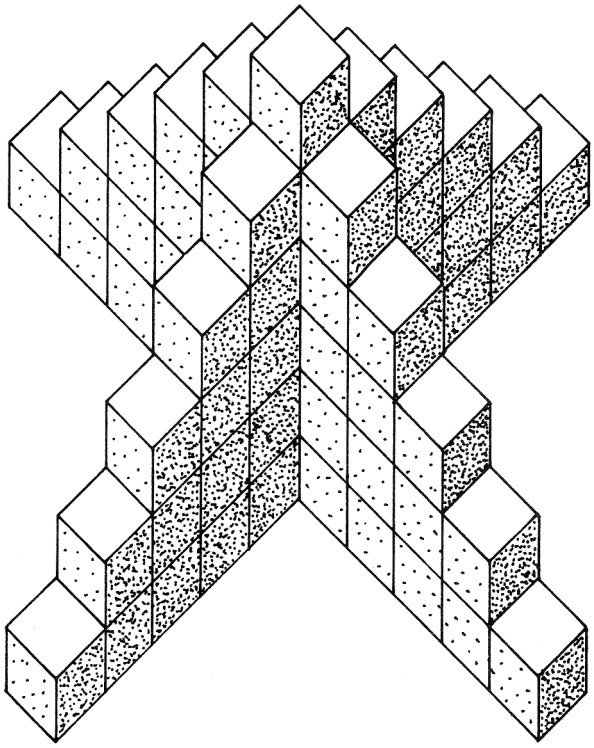
Skeleton Tower



1. How many cubes are needed to build this tower?
   Show your calculations

2. How many cubes are needed to build a tower like this, but 12 cubes high?
   Explain how you figure out your answer.

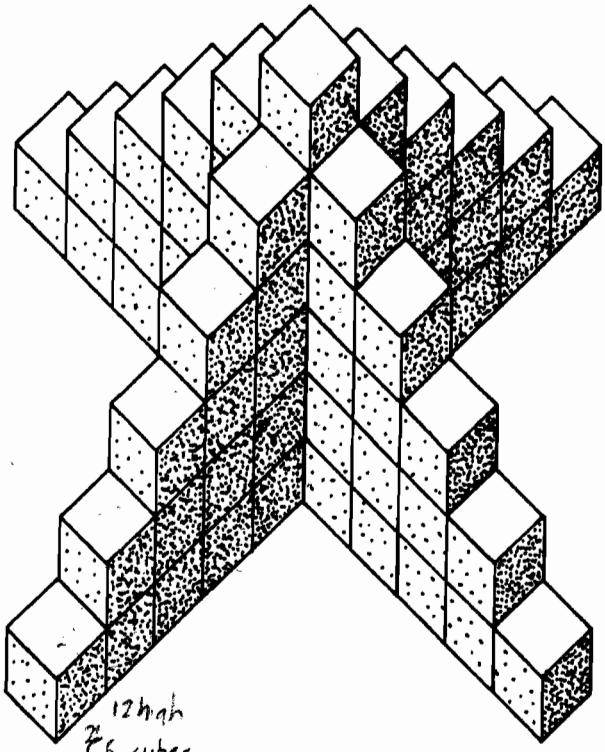
Skeleton Tower (continued)

3. How would you calculate the number of cubes needed for a tower *n* cubes high?

Skeleton Tower	Rubric	
	Points	Section points
1. Gives correct answer: **66** *Partial credit:* Adds together the numbers 1, 5, 9, 13, 17 and 21 with no more than one error.	2 (1)	2
2. Gives correct answer: **276** Gives a correct explanation such as: Each layer has 4 more cubes than the layer above. Adds together 12 layers. Alternatively: May add the number of cubes in one wing of the tower, and then multiply by 4. The number of cubes on the 12 layers are 1,5,9,13,17,21,15,29,33,37,41,45	1 2 or 2 1	4
3. Gives correct answer: **$2n^2 - n$** *Partial credit*: Finds the number of cubes for at least the first 4 towers to try to find the pattern. 1, 6, 15, 28, 45, 66 ….. Provides a correct explanation showing how the formula can be found.	2 (1) 2	4
Total Points		**10**

Skeleton Tower

T1



15
× 4
60
+ 6
66

12 high
66 cubes
× 4
264
+ 12

276 cubes

1. How many cubes are needed to build this tower?
   Show your calculations

66 cubes

2. How many cubes are needed to build a tower like this, but 12 cubes high?
   Explain how you figure out your answer.

276 cubes

for this problem only 1 cube is alone on top with a single row of cubes going down center. However 11 rows of cubes aren't attached to middle row. From the 5 rows already shown add another row with 1 extra up to 11 rows to 66 cubes multiply by 4 to get 264 cubes and add the 12 middle row to get 276 cubes

Skeleton Tower (continued)

3. How would you calculate the number of cubes needed for a tower n cubes high?

no of cubes in each wing is $1+2+3+4+\cdots(n-1)$ which is $\frac{n(n-1)}{2}$

There are 4 wings so the number of cubes is $\frac{4n(n-1)}{2}$

This does not include the number of cubes down the centre

so the complete formula for the number of cubes is

$$\frac{4n(n-1)}{2} + n = 2n(n-1) + n = 2n^2 - 2n + n$$

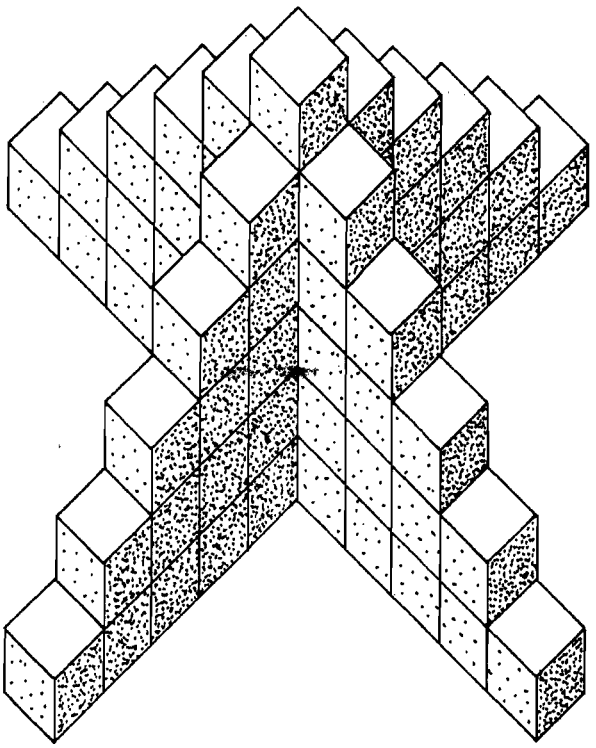
$$= 2n^2 - n$$

check $n = 6 \quad 2 \cdot 6^2 - 6 = 72 - 6 = 66$ ✓

$n = 12 \quad 2 \cdot 12^2 - 12 = 288 - 12 = 276$ ✓

Skeleton Tower

T2



1. How many cubes are needed to build this tower? 66
   Show your calculations

each side w/o center post has height (1+2+3+4+5) = 15
4 sides = 60
+ 6 cubes in center = 66

2. How many cubes are needed to build a tower like this, but 12 cubes high? 276
   Explain how you figure out your answer.

12 + 4 (1+2+ ··· 11) 12 + 4((12·5)+6) = 12 + 4(66) = 12 + 264 = 276

(I found the formula first)

T2

Skeleton Tower (continued)

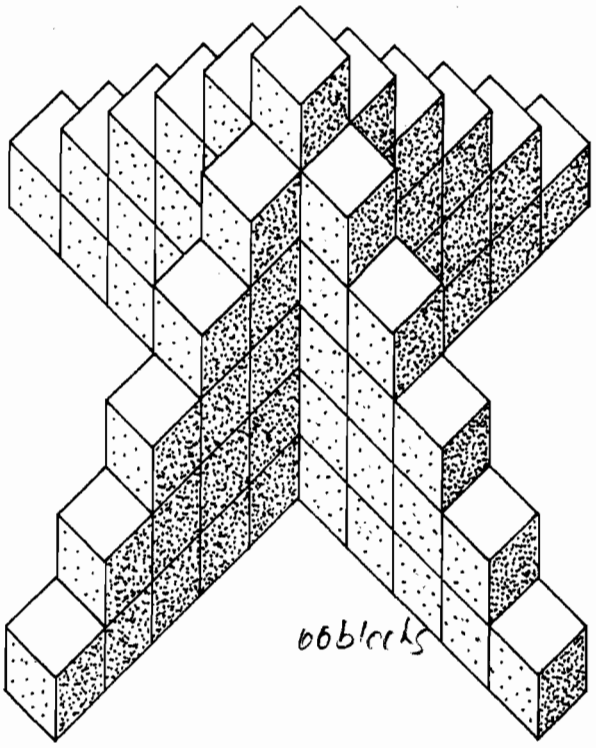
3. How would you calculate the number of cubes needed for a tower *n* cubes high?

$n + 4(1+2\cdots(n-1))$

4 sides w/o center would always be $1+2+3+4\cdots$ until the number just before $n:(n-1)$., the center post is the highest, so the height n of the tower determines how tall the center post is

Skeleton Tower

T3



1. How many cubes are needed to build this tower?
   Show your calculations

66 cubes

15×4 + 5 + 1 = 66

2. How many cubes are needed to build a tower like this, but 12 cubes high?
   Explain how you figure out your answer.

276 cubes

x = height of tower ; Formula each wing # of blocks = $\frac{(x-1)x}{2}$

Total blocks 4 wings : $\frac{4(x-1)x}{2} = 2x(x-1)$

center tower blocks : x

Total # of blocks of entire tower = $2x(x-1) + x =$

$2x^2 - 2x + x =$

$2x^2 - x$

$2(12)^2 - 12$

$288 - 12 = 276$

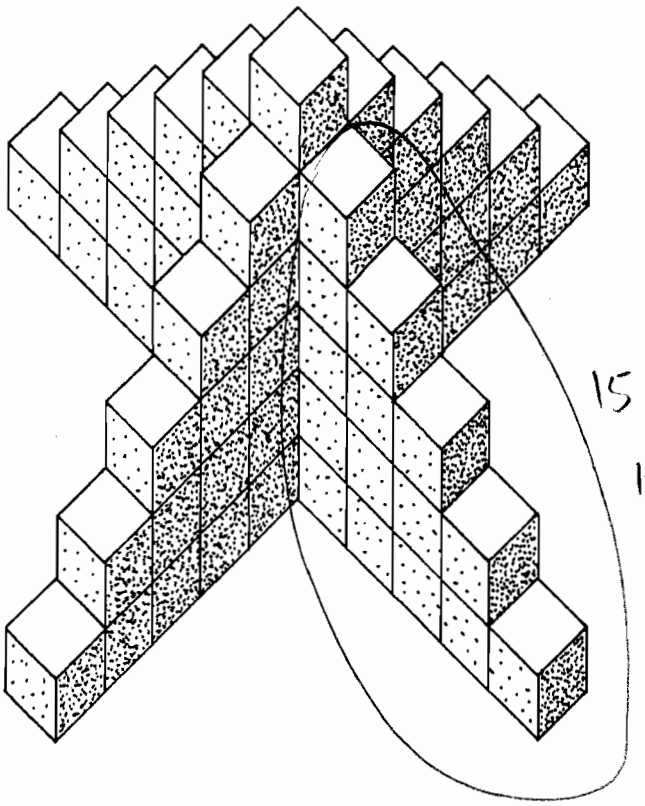
Skeleton Tower (continued)

3. How would you calculate the number of cubes needed for a tower n cubes high?

$2 \cdot n^2 - n$

Skeleton Tower

T4



1. How many cubes are needed to build this tower?
   Show your calculations

66 cubes

15 × 4 = 60

69 − 6 = 66

2. How many cubes are needed to build a tower like this, but 12 cubes high?
   Explain how you figure out your answer.

276 cubes

1+2+3+4+5+6+7+8+9+10+11=66

(66)(4)=264

264+12=276

Skeleton Tower (continued)

3. How would you calculate the number of cubes needed for a tower n cubes high?

$1+2+3+\ldots+(n-1) = \frac{n(n-1)}{2}$

$4\left(\frac{n(n-1)}{2}\right) + n$

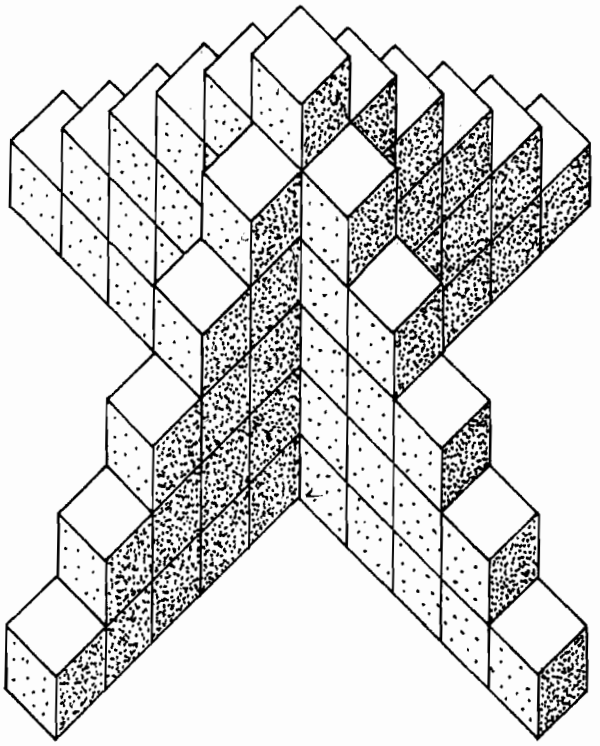
$4\left(\frac{n^2-n}{2}\right) + n$

$2n^2-2n+n$

$\boxed{2n^2-n}$

Skeleton Tower

T5



1. How many cubes are needed to build this tower?
   Show your calculations

66 cubes

$4(5+4+3+2+1)+6$

↑ # of sides ↑ sides ↑ middle column

2. How many cubes are needed to build a tower like this, but 12 cubes high?
   Explain how you figure out your answer.

270 cubes

I used the formula $4(11+10+9+8+7+6+5+4+3+2+1)+12$.

$= 4(66)+6 = 264+6 = 270$

T5

Skeleton Tower (continued)

3. How would you calculate the number of cubes needed for a tower n cubes high?

I would use the formula $4\left(\frac{n(n-1)}{2}\right)+n$

sides of tower

# of cubes center column height

# of cubes per side $1+2+3+4+\ldots+(n-1)$