

Proofs Of The Pythagorean Theorem?

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

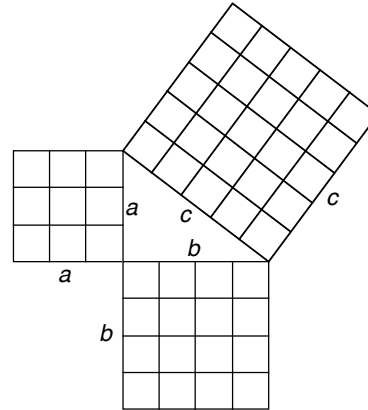
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

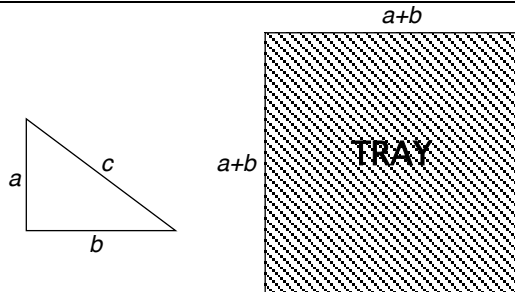
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



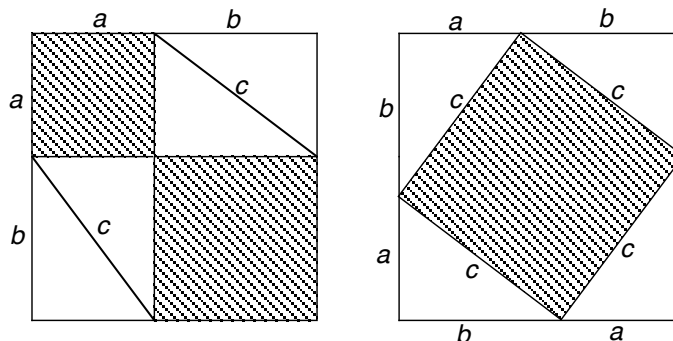
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

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Since these areas are equal

$a^2 + b^2 = c^2$

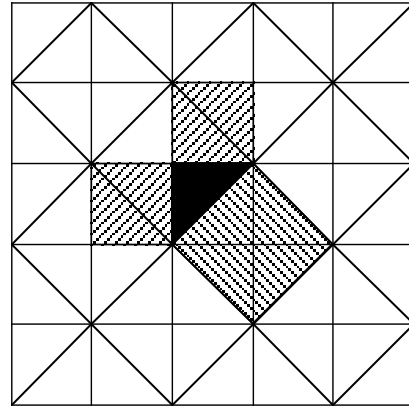


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side-the hypotenuse.



The best proof is attempt number _____

This is because

My criticisms of the others are

Proofs of the Pythagorean Theorem		Rubric	
		Points	Section points
Gives correct answer: The best proof is attempt number 2 .		3	10
Explains that attempt number 2 works for any size of right triangle.		2	
Explains that attempt number 1 only shows that the theorem works for a 3, 4, 5 triangle.		2	
Explains that attempt number 3 only shows that the theorem works for isosceles right triangles.		3	
Total Points			10

Proofs Of The Pythagorean Theorem?

T1

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

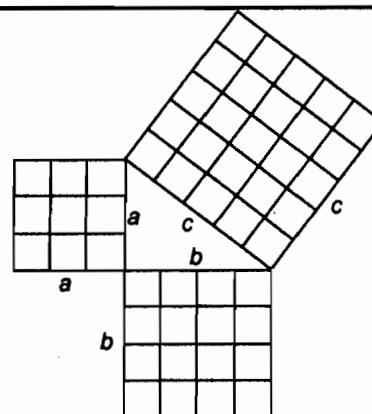
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

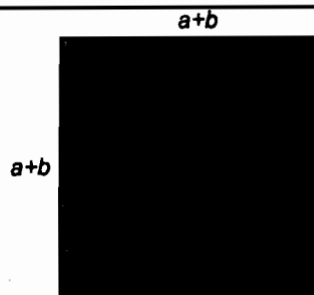
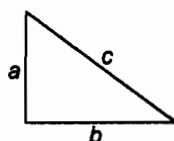
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



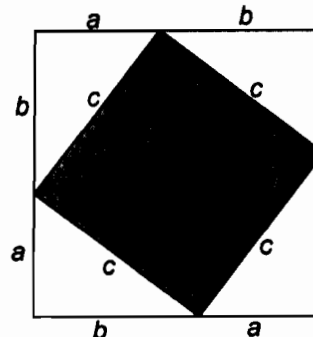
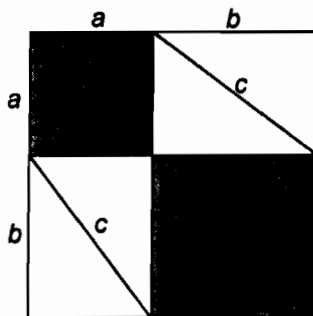
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Since these areas are equal.

$a^2 + b^2 = c^2$

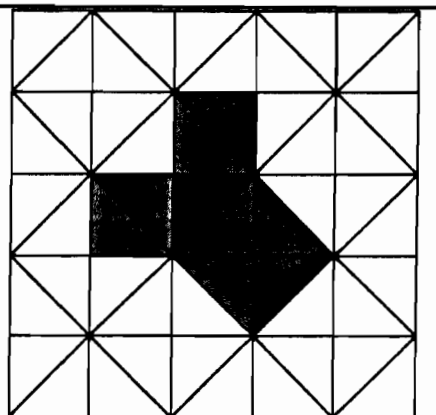


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number 2

This is because

It proves that the Pythagorean theorem is
always true, no matter what a, b, and c are
By arranging the triangles in different
way it accurately demonstrates that
 $a^2 + b^2 = c^2$ is always true if

My criticisms of the others are.

Attempt 1 only proves that $a^2 + b^2 = c^2$ if $a=3$,
 $b=4$ and $c=5$. Attempt 3 only proves that
 $a^2 + b^2 = c^2$ if a and b are congruent and there
is a right angle.

Proofs Of The Pythagorean Theorem?

T2

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

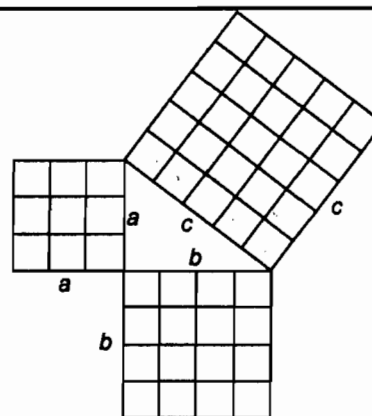
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

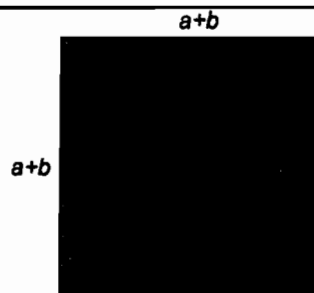
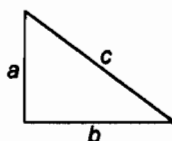
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

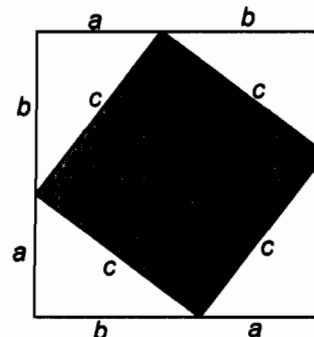
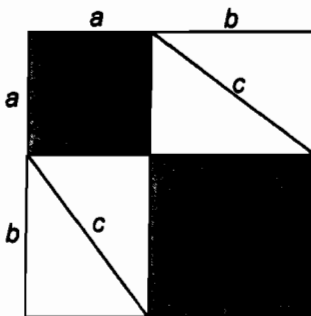
Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$. In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal
 $a^2 + b^2 = c^2$

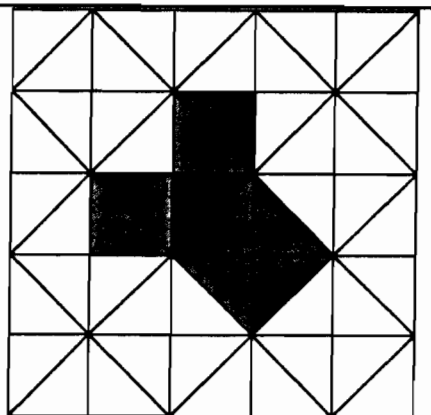


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number 2

This is because

I can easily understand and it works for every trio
of numbers, not just 3,4,5. We can see the difference
by the shading and labeling. We can also 'see' the
squares.

My criticisms of the others are.

- 1 $a^2 + b^2 = c^2$ But it doesn't always equal
 $3^2 + 4^2 = 5^2$ It can be $5^2 + 12^2 = 13^2$ or other
combinations
3. It just doesn't make sense.

Proofs Of The Pythagorean Theorem?

T3

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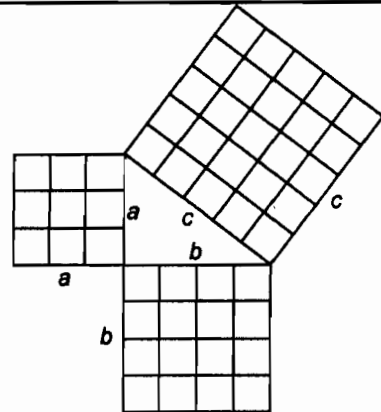
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Draw squares on the three sides as shown.

Divide these squares into smaller squares.

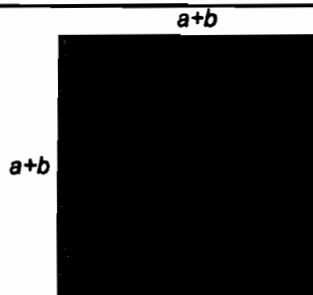
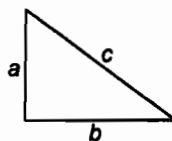
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

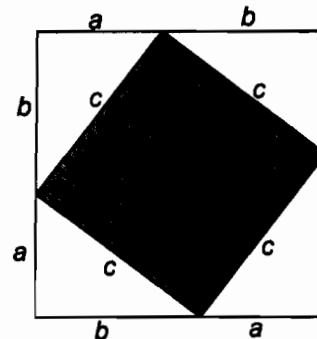
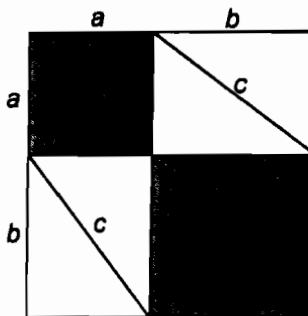
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You can arrange the triangles into the tray in two different ways as shown here.

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Since these areas are equal
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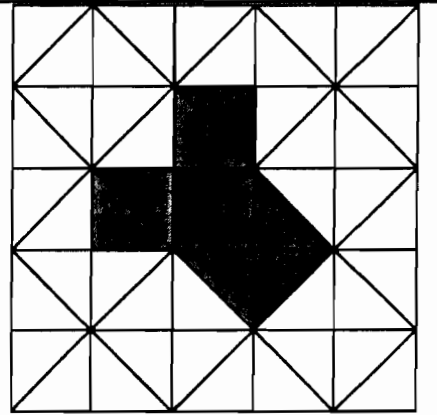


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number 2

This is because

It doesn't make any specific assumptions about the triangle
other than the fact that it is a right triangle. Sides a and b are
not necessarily congruent, and they don't state that they're in
a specific ratio

My criticisms of the others are.

Attempt 3 doesn't work with a totally arbitrary right triangle
Instead, it works with an isosceles right triangle, and this has only
proved the theorem for isosceles right triangle. Attempt one makes
the assumptions that the sides are in a 3:4:5 ratio, which isn't
always true for right angles

Proofs Of The Pythagorean Theorem?

T4

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

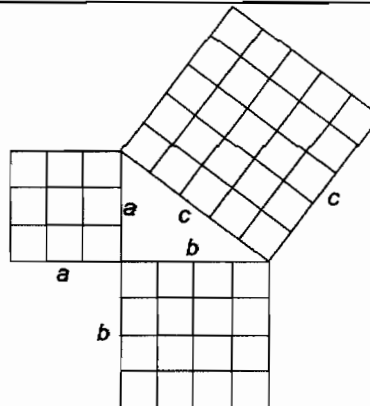
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

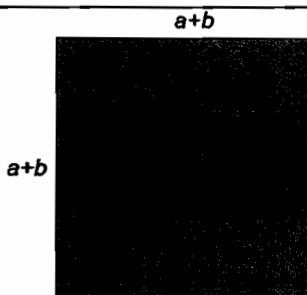
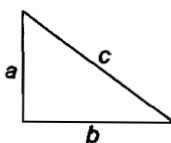
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



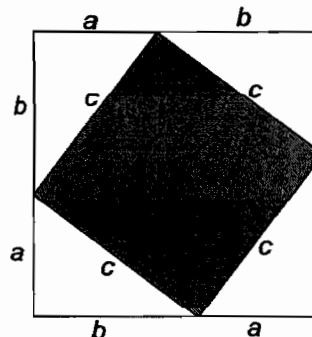
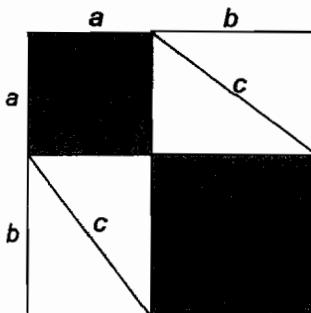
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In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

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Since these areas are equal

$a^2 + b^2 = c^2$

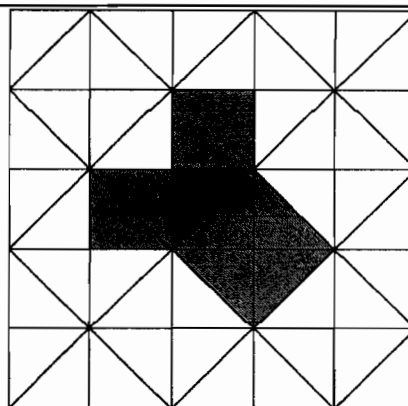


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number 2

This is because

It doesn't make any specific assumptions about the triangle other than the fact that it has a right triangle sides a and b are not necessarily congruent, and they don't state that that they're in a specific ratio, only $a^2 + b^2 = c^2$

My criticisms of the others are.

Attempt 3 only works for an isosceles right triangle not for any right triangle.

Proofs Of The Pythagorean Theorem?

T5

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

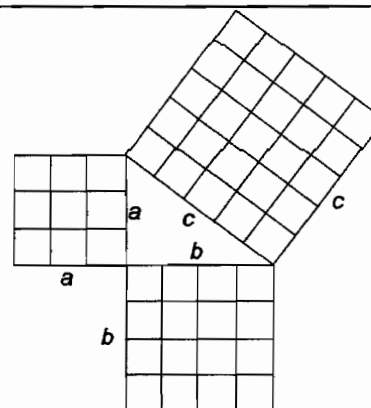
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

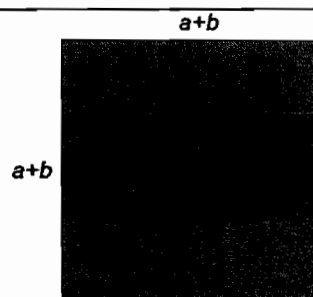
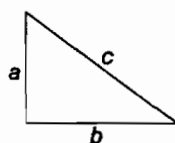
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



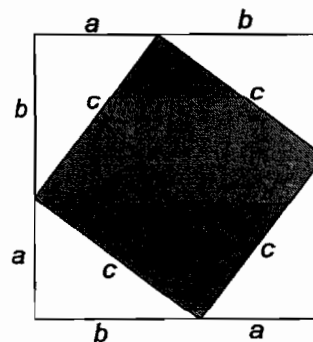
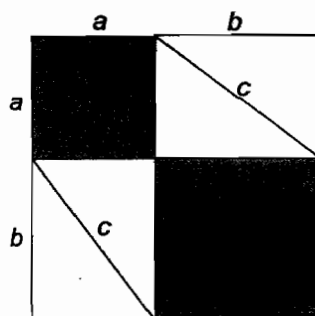
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Since these areas are equal

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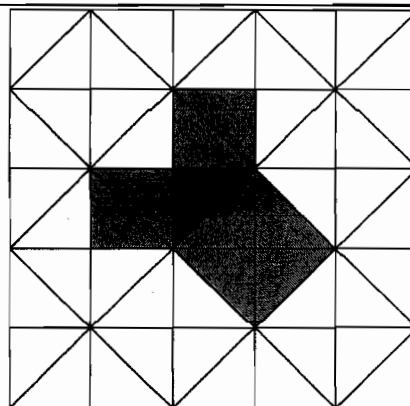


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number

2

This is because

The area of the 4Δ + the area of the black square
= the entire square $4(\frac{1}{2}ab) + c^2 = (a+b)^2$

$$2ab + c^2 = a^2 + 2ab + b^2$$

$$c^2 = a^2 + b^2$$

You don't have to assume any side ratios and
a and b can be any number as long as there
is a right angle

My criticisms of the others are.

For attempt 1, a, b and c have to be whole numbers

You have to know the ratio of the sides in order for
the squares to be able to be split into smaller whole

squares. For attempt 3, the right triangle must be
45-45-90. It is isosceles and does not apply to
any other types of triangles.