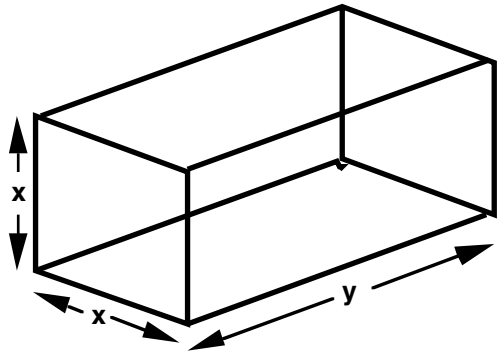
Fearless Frames

Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

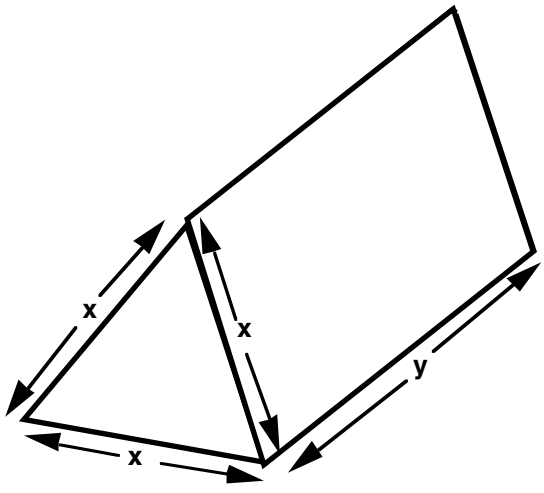
Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.



2. The client changes his mind!

He asks for a container which is a prism with a cross section which is an equilateral triangle

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



Fearless Frames: (continued)

3. What advice do you think Fearless Frames should offer to this customer?
   Show all your calculations.

Fearless Frames	Rubric	
	Points	Section points
Shows that the volume of the prism $V = x^2y$. The perimeter of the prism $P = 8x + 4y = 60$ $y = 15 - 2x$ $V = x^2(15 - 2x)$	3	
The graph of V against x shows that as x increases from 1 to 5 the volume increases, and than decreases for values of x from 5 to 7. V is max when x = 5.	2	
Alternatively May make a list showing the values x = 4 and volume 112 x = 6 and volume 108 When x = 5, y = 5 and V = 125	or	
States that for P = 60 meters, the maximum volume is 125 cubic meters.	2	5
Shows that the height of the equilateral triangle is $\sqrt{3}x/2$. The volume of the prism (V) = $\sqrt{3}x^2y/4$ The perimeter of the prism (P) = $6x + 3y = 60$ $y = 20 - 2x$ $V = \sqrt{3}x^2(20 - 2x)/4$ V is maximum when $x = y = 6^2/_3$ (accept values 6 – 7) For perimeter 60 meters, the maximum volume is 128 cubic meters. Accept vales 124 - 128	4	4
Advise the customer that, using 60 meters of tubing, a container with a cross section which is an equilateral triangle holds a little more than one which is a square.	1 ft	1
Total Points		**10**

Fearless Frames

T1

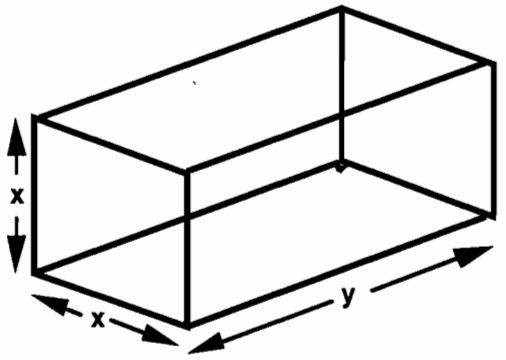
Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.



$P = 4y + 8x = 60$

$y + 2x = 15$

$y = 15 - 2x$

$V = x^2 y = x^2(15 - 2x)$

$x = 4 \quad y = 15 - 8 = 7$

$V = 16 \cdot 7 = 112$

$x = 5 \quad y = 15 - 10 = 5$

$V = 25 \cdot 5 = 125$

$x = 6 \quad y = 15 - 12 = 3$

$V = 36 \cdot 3 = 108$

So when $x = 5$ and $y = 5$ the volume is at its highest at $125\ m^3$

T1

Fearless Frames: (continued)

2. The client changes his mind!

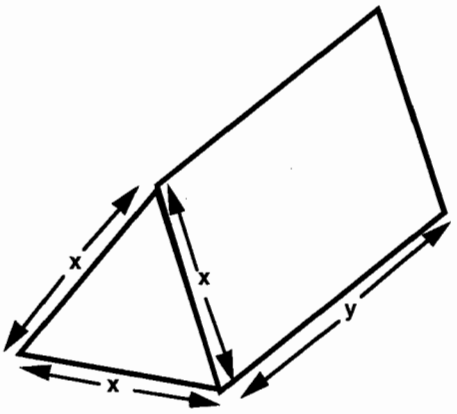
He asks for a container that is a prism with a cross section which is an equilateral triangle.

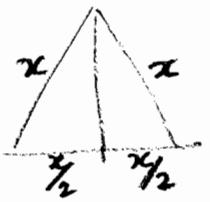
Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

$P = 60 = 6x + 3y$

$20 = 2x + y$

$y = 20 - 2x$





$a^2 + b^2 = c^2$

$a^2 + \left(\frac{x}{2}\right)^2 = x^2$

$a^2 = x^2 - \frac{x^2}{4}$

$= \frac{3x^2}{4}$

$a = \frac{\sqrt{3}(x)}{2}$

$Vol = \frac{x}{2} \cdot \frac{\sqrt{3}(x)}{2} y$

$= \frac{\sqrt{3}x^2(20 - 2x)}{4}$

- When $x = 5$, $y = 20 - 2x = 10$

$= \frac{\sqrt{3} \cdot 25 \cdot 10}{4} = 108.25$

- When $x = 6$, $y = 20 - 12 = 8$

$V = \frac{\sqrt{3} \cdot 36 \cdot 8}{4} = 124.7$ bigger.

- when $x = 7$, $y = 20 - 14 = 6$

$V = \frac{\sqrt{3} \cdot 49 \cdot 6}{4} = 127.3$ bigger

- when $x = 8$

$\frac{\sqrt{3} \cdot 64 \cdot 4}{4} = 110.8$ smaller

- when $x = 6.5$

$y = 20 - 13 = 7$

$Vol = \frac{\sqrt{3} \cdot 42.25 \cdot 7}{4} = 128.06\ m^3$ highest between $x = 6$ $x = 7$

when $P = 60$

$V = 128\ m^3$

3. What advice do you think Fearless Frames should offer to this customer? Show all your calculations.

There's not much in it. When $x = y$, the rectangle holds $125 m^3$ and the prism holds $128.06 m^3$. The prism holds a little more as it would take a long time and patience to work out the correct dimensions same where between 6 and 7

Fearless Frames

T2

Fearless Frames Inc makes metal frames for containers.

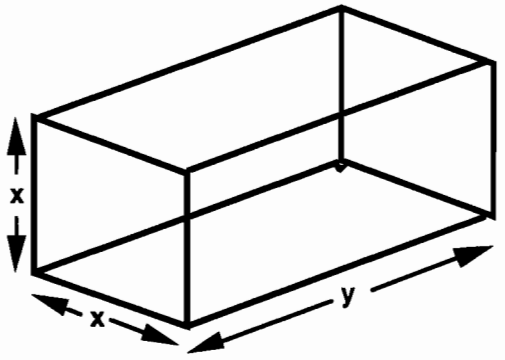
1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

Max volume = 125 in^3

The company has only 60 meters of suitable metal tubing in stock. Dimensions = 5 x 5 x 5

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.



X	volume
1	13
2	99
3	81
4	112
5	125
6	108

volume = $X^2 \times y$

$X^2 \times (-2x+15)$

$-2x^3 + 15x^2$

$-2(5)+15$

$-10+15$

$y = 5$

$4x+4x+4y-60$

$60x-4y-60$ $4y$

$8x-60$ $4y$

4 9

$y = -2x+15$

Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

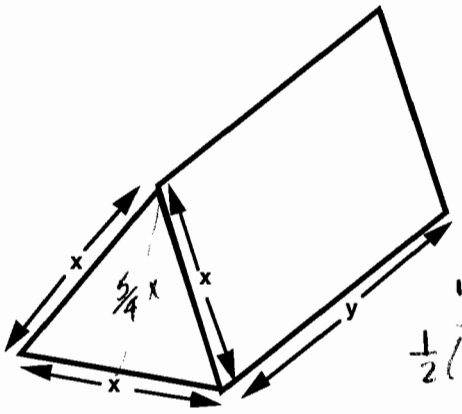
Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

$x^2 + \left(\frac{1}{2}x\right)^2 = h^2$

$x^2 + \frac{1}{4}x^2 = h^2$

$\sqrt{\frac{5}{4}x^2} = \sqrt{h^2}$

$\frac{5}{4}x = h$



If $x = 5, y = 10$

$\frac{1}{2}(5)\left[\left(\frac{5}{4}\right)5\right].10$

$= 15.6\ 25.10$

$= 156.25\ m^3$

If $x = 6, y = 8$

$\frac{1}{2}(6)\left[\frac{5}{4}6\right].8 = 180\ m^3$

length

$2(3x) + 3y = 60$

$6x + 3y = 60$

$2x + y = 20$

$y = 20 - 2x$

Volume $= \frac{1}{2}bh \cdot y$

$\frac{1}{2} \times \left(\frac{5}{4}x\right) \cdot y$

If $x = 4\ y = 12$

Vol $\frac{1}{2}(4)\left(\frac{5}{4}(4)\right).12 = \frac{1}{2}(20).12$

$= 120\ m^3$

If $[x = 7\ y = 6]$

$\frac{1}{2}(7)\left[\left(\frac{5}{4}\right)7\right].6$

$= 30.625.6$

$= \boxed{183\ 75\ m^3}$

3. What advice do you think Fearless Frames should offer to this customer? Show all your calculations.

The prism container has more volume. If the customer wants the maximum volume, the dimensions (x, y) would be very close to give the most space. If the customer likes larger containers, he should go with the rectangular prism

Fearless Frames

T3

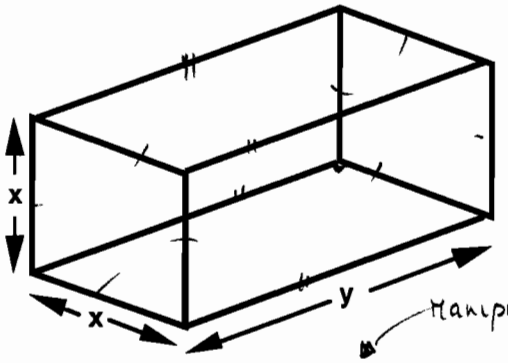
Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.



Find largest if x is a whole number

Area of base $= x^2$

$8x + 4y = 60$

$\frac{8x}{4} + \frac{4y}{4} = \frac{60}{4}$

$2x + y = 15$

$y = 15 - 2x$

Manipulative variable

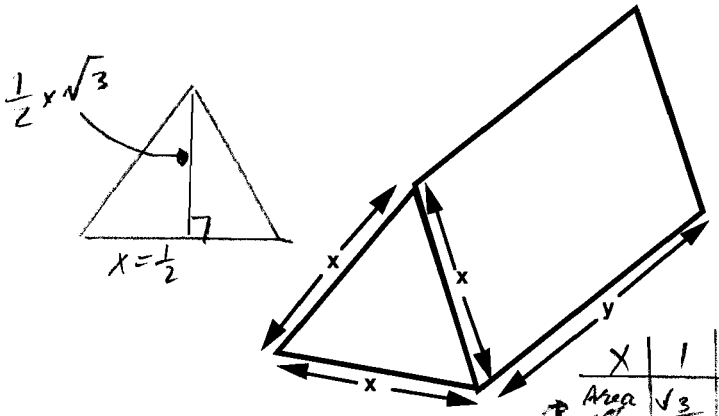
x	1	2	3	4	5	6	7
Area of Base	1	4	9	16	25	36	49
y	13	11	9	7	5	3	1
Volume	13	44	81	112	125	108	49

Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



X	1	2	3	4	5	6	7	8
Area of Base	$\frac{\sqrt{3}}{4}$	$\sqrt{3}$	$\frac{9}{4}\sqrt{3}$	$\frac{25}{4}\sqrt{3}$	$16\sqrt{3}$	$9\sqrt{3}$	$\frac{49}{4}\sqrt{3}$	$16\sqrt{3}$
y	18	16	14	12	10	8	6	4
	$\frac{9}{2}\sqrt{3}$	$16\sqrt{3}$	$\frac{63}{2}\sqrt{3}$	$48\sqrt{3}$	$\frac{125}{2}\sqrt{3}$	$72\sqrt{3}$	$\frac{147}{4}\sqrt{3}$	$\frac{81}{2}\sqrt{3}$
	$4.5\sqrt{3}$		$31.5\sqrt{3}$		$62.5\sqrt{3}$		$73.5\sqrt{3}$	

Area of base $= \frac{1}{2}bh$

$= \frac{1}{2}(x)(\frac{1}{2} \times \sqrt{3})$

$= \frac{1}{4} x \sqrt{3}$

Area of Base $= \frac{\sqrt{3}}{4} x^2$

$6x + 3y = 60$

$\frac{3}{3}$

$2x + y = 20$

$y = 20 - 2x$

$V = (\text{area})(y)$

$\sqrt{3}\frac{9}{4} \cdot 14 = 63\sqrt{3}$

3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

For both containers, it's best they choose the highest value for x, while y is still a whole number.

Fearless Frames

T4

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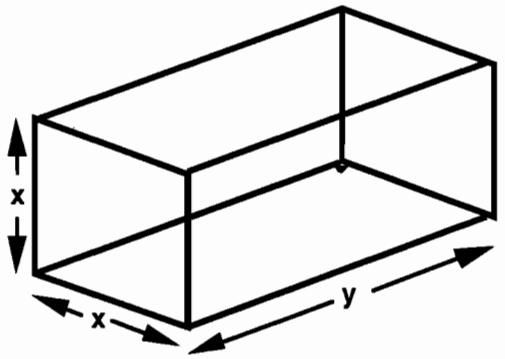
The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

length:



$8x + 4y = 60$

$2(4x) + 4y = 60$

$2x + y = 15$

$y = 15 - 2x$

Volume $= x^2 \cdot y = x^2(15 - 2x)$

if $x = 4$ $y = 7$

Volume $= 4^2 \cdot 7 = \boxed{112\ m^3}$

if $x = 5$, $y = 5$ but not sure if x can $= y$

$5^2 \cdot 5 = \boxed{125\ m^3}$

if $x = 6$, $y = 3$

$6^2 \cdot 3 = \boxed{108\ m^3}$

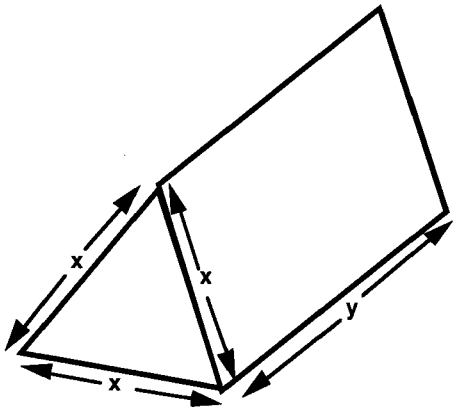
Fearless Frames: (continued)

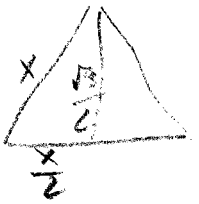
2. The client changes his mind!

   He asks for a container that is a prism with a cross section which is an equilateral triangle.

   Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

Max Volume
Dimensions





$3x+3x+3y=60$

$6x+3y=60$

$\frac{6x-60}{-3} = \frac{+3y}{+3}$

$30-2y=y$

Volume $= \frac{x \cdot h}{2} \cdot y =$

$V = \frac{x\left(\frac{\sqrt{3}\cdot x}{2}\right)}{2} \cdot (-2x+30)$

$x\left(\frac{\sqrt{3}}{2}\right) \cdot 2(-x+15)$

$V: \frac{\sqrt{3}x2-(-x+5)}{2}$

$-\sqrt{3}x^3+15\sqrt{3}\cdot x^2=20$

$-\sqrt{3}(10)^3+15\sqrt{3}\cdot(10)^2=20$

$\cdot 100 = 20$

3. What advice do you think Fearless Frames should offer to this customer?
   Show all your calculations.

Fearless Frames

T5

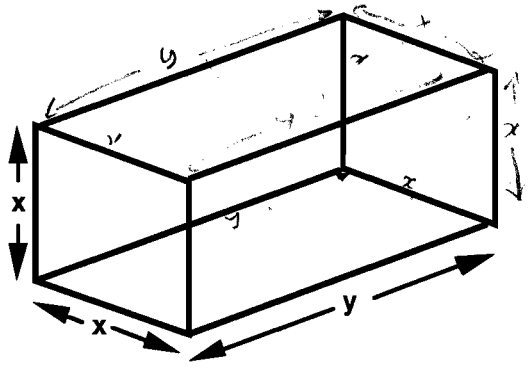
Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.



X	V
1	13
2	44
3	81
4	112
5	125
6	108

X	V
4.5	121.5

$8x + 4y = 60$

$2x + y = 15$

$y = 15 - 2x$

$V = x^2 y$ = maximum

$V = x^2(15-2x) = 15x^2 - 2x^3$

$x = 5m$ $y = 15-10$

$y = 5m$

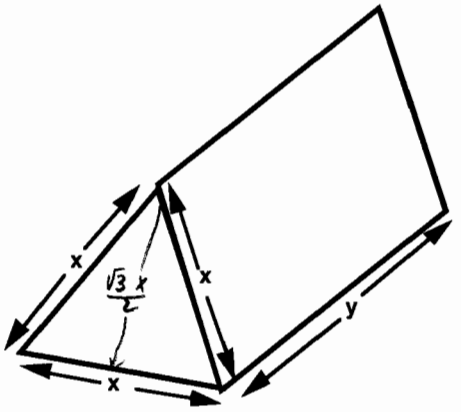
$V = 125 m^3$

Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



$0 < x < 10$

x	$x^2(10-x)$
2	32
3	63
4	96
5	125
6	144
7	147
8	128
6.5	147,875
6.7	148,137
6.8	147,968

$6x + 3y = 60$

$2x + y = 20 \quad y = 20 - 2x$

$V = \frac{\sqrt{3}}{4} x^2 y \quad \frac{\sqrt{3}}{2} x^2 (10 - x)$

Maximize $x^2(10-x)$

$x = 6.7$ m $\quad y = 20 - 13.4$

$y = 6.6$ m

$V = \frac{\sqrt{3}}{2} (6.7)^2 (10 - 6.7)$

$V = \frac{\sqrt{3}}{2} (44.89)(3.3)$

$V = 128.29 \text{ m}^3$

3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

It is better to use a triangular frame than a square frame, because a triangular frame gives you a bigger volume while using the same amount of wiring.